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BIOTIC CEMENTATION IN CORAL REEFS

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Any discussion concerning coral reefs might readily take into account the difference between "facts" and "fancies." Much of the writing concerning coral reefs may fairly be said to belong under the latter designation, being based upon charted details with little other knowledge of the exact nature of the reef structures concerned than their topography. Much, even of Darwin's work on reefs, is based on charted details. There are, consequently, a number of assumptions which have become so permanently embedded in the literature as to pass muster practically without scrutiny, and it seems best to inquire as to what is actually meant by the term "coral reef." A reef, at least in the sense of a structure of seas or oceans, is defined as a ridge of rock or sand whose upper surface is at or near the surface of the water, and again, as within 6 fathoms of the surface, while a bank has the upper surface not less than 6 fathoms below the surface. The prefix "coral" may be taken literally, or it may mean simply "calcareous." In the literature, the expression "coral reef" is held to be a reef, containing more or less of coral growth forms (corolla) and whose mass has resulted from coöperative (or symbiotic) coral growth. The corals whose growth has provided the stony structure of the reef are spoken of a "reef-forming corals," usually without particular indication as to any difference of behavior or structure between them and non-reef-forming corals. Furthermore, coral reefs, in the literature, at least, are divided among fringing, barrier, and atoll coral reefs. In any case, coral reefs, as such, and without other special designation, meant to the earlier writers, solid structures or at least with solid framework, built up by the cooperative growth of coral colonies, whose skeletons became their chief, if not their almost exclusive, component, and whose physiographic (and ecologic) form was one of three types of zonation or belting.

In the Darwinian theory, the coral constituent, by its upward growth, is supposed to keep reasonable pace with the slowly subsiding reef substratum to convert fringing coral reef into barrier coral reef, and barrier

coral reef, finally, if subsidence be sufficiently long continued, into atoll coral reef. This universally assumed power has never been critically tested, or even its likelihood, other than in hypotheses, carefully examined.

The coral reefs of Darwin, Dana, Alexander Agassiz, etc., then, are coherent structures made solid and wave-resistant through coöperative growth in the biotic sense and not aggregates of calcareous materials, rubble or sand, heaped up by waves or currents but incapable of increment by the self-multiplication and mutual coherence of their parts. This distinction is to be insisted upon with some considerable degree of vehemence, since various coral reefs and coral islands of the western and northwestern coasts of Australia and of the Sunda and other seas of the East Indies, and possibly others undistinguishable on maps and charts, are not at all to be classified with the coral reefs described and discussed by Darwin, Dana, etc., nor is either their origin or their development to be attributed to cooperative biotic growth. The coral reef theory of Darwin cannot possibly be either demonstrated or disproved simply by the study of reefs as delineated geographically.

There is so little coherence to be found between contiguous coral skeletons which make up coral reefs of the Darwinian types that the solidity of these structures, so important in preserving their form as they are built up, must be sought elsewhere. The rosette or rosulate life (or growth) forms of the reef-forming corolla, when not bound together and welded into a resistant reef-mass, leave the adult corals to die, to be broken away and cease to form any part of a typical coral reef, exposed as it is to violent onslaught of wave-forces unless they or their fragments are caught and held together in some fashion or other. The same thing is true of "boss" and "brain" life-forms of corolla, but for the "staghorn" types of life-form, no one has even hinted at any possible coherence. This process of the disintegration of coral veneer is to be observed on old reefs, where cementation is no longer going on. Inorganic cementation, such as is at times assumed to accomplish the welding, cannot be considered, since it has not been demonstrated as occurring in those parts of the reef, viz., the outer rim and flank, where growth alone can be carried on. Cementation by other symbionts than the corals is to be looked for. It is readily observed on rims and flanks of growing reefs, and its effect is to be verified in the older parts of elevated reefs which have been eroded by wave action into cuts and caverns which show extensive sections of reef structure. By far the most, and one may say the only, effective cementation is due to certain crustaceous corallines or nullipores. In sections of emergent reefs, such as are to be seen on the windward shores of Tongatabu and on those of Oahu, the various coral constituents and various nullipores, shells, etc., are all surrounded and bound together by the thinner or thicker crusts of the nullipores into a compact mass, or at least into a massive skeleton, whose interstices finally become

filled with sand and rubble of corals, shells and foraminifera. The same structure is revealed in cores from borings, such as those made at Funafuti. It is the biotic cementation by the nullipore which sets off the coral reef (or island) of Darwin and his successors from other calcareous reefs, and it is the nullipore, therefore, which controls and shapes the reef from its origin to its final form. Since cementing and even what may properly be called reef-forming nullipores may be the exclusive constituents of a so-called "coral reef," and since they are, as a group of organisms, exempt from the depth limitation of "reef-forming corals," the only observation of Darwin's hypothesis (hardly a true theory) is eliminated; and his assumption of interconvertibility and the consequent assumption of subsidence during reef development are unnecessary.

THE QUANTITATIVE THEORY OF SEX AND THE GENETIC CHARACTER OF HAPLOID MALES

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Introduction.—The foundations were laid for a quantitative theory of sex by the brilliant pioneer work of Goldschmidt and Bridges.

1. Goldschmidt was led to formulate a quantitative theory of sex by his studies of the genetics of the gipsy moth. He found that when different geographical races were crossed, as for example European and Japanese, individuals of mixed sexual character were often produced which he termed intersexes. Sometimes these individuals were predominantly female, sometimes predominantly male; sometimes their change from the expected sex character was so complete as to amount to sex reversal and only unisexual broods were obtained, exclusively male or exclusively female.

Oftentimes reciprocal crosses gave unlike results, one cross producing a normal result with equal numbers of males and females, whereas the reciprocal cross would produce only females and male intersexes, or only males and female intersexes. In extreme cases the intersexes would be transformed into the opposite sex, so that unisexual broods resulted.

Goldschmidt explained these results by assuming that maleness and femaleness are determined by genes borne in different chromosomes and that the strength or potency of these genes varies from race to race. Crosses bring together male and female genes of different relative potency and this upsets the usual sex balance and produces intersexes.

2. Bridges and his co-workers reached conclusions somewhat similar

from their studies of triploid individuals of Drosophila and of their descendants. Triploids inevitably produce unbalanced chromosome conditions in a considerable proportion of their descendants, and it was from a study of these that Bridges was able to make a more complete and certain analysis of the influence on sex of each set of chromosomes than had been attempted by Goldschmidt.

The method of sex determination is also different in Drosophila from the method found in the gipsy moth (Lymantria), the form studied by Goldschmidt, and this led to minor differences in their conclusions.

Bridges found that the X-chromosome of Drosophila has a female influence which is superior to the male influence of the autosomes (II, III, and IV). Accordingly any multiple of XA (letting A stand for a complete set of autosomes) will be female. Thus 2X2A is an ordinary diploid female, 3X3A is a triploid female and 4X4A is a tetraploid female, all of which have been experimentally produced and identified. A few individuals have also been produced which were haploid (XA) in certain mosaic segments of the body which was elsewhere diploid. These haploid segments appear to be female also, as they produce no sex-combs, a male character.

But in Drosophila the female influence of X is less than the male influence of two sets of autosomes, since an individual X2A or XY2A in constitution is a male. But a 2X3A individual is an intersex in which male and female characters both find expression. Hence $2X = 3A$, and $X = 1.5A$ would be a quantitative statement of the relative potency of the respectively female and male influences of X and A.

No individual genes influencing sex have been found by students of Drosophila in any of the chromosomes, though search has been made for them, and the failure to find them is significant in view of the very complete linkage studies made on this species.

In the quantitative theory of sex as formulated by Goldschmidt and adopted by Winge and Hartmann, it is assumed that there are specific and separate genes for maleness and femaleness and that the relative strength or valence of these determines what the sex shall be in a particular organism.

But the existence of such genes has never been conclusively demonstrated, as should be possible by linkage studies if they really existed, as postulated, in particular chromosomes. Their status is purely hypothetical and, I think, superfluous.

I propose a simplified quantitative theory of sex, in accordance with which it is unnecessary to suppose that there are specific genes for maleness and femaleness, but sex differences are merely plus and minus variations in a single scale of gradations in sexual character.

Haploid Sex Determination.—Sex is exemplified in its simplest form in a

tendency of two cells to unite to form a new individual. The uniting cells must differ from each other physiologically even when they are morphologically alike. The primary difference is perhaps in character of metabolism, as suggested many years ago by Geddes and Thompson. A more vigorously growing type, predominantly anabolic in its metabolism, can be distinguished from a less vigorously vegetative type predominantly katabolic in its metabolism. Blakeslee (1904) when he discovered the existence of sexual differentiation among bread-moulds and related fungi distinguished one type as +, the other as −. He found that sexual union could occur only between a+ and a− strain. Among the algae a similar differentiation exists, as shown by Schreiber (1925) in the case of Gonium. The swarm spores (gametes) which unite in pairs are all motile and morphologically alike but fall into two classes + and − in sexual reaction.

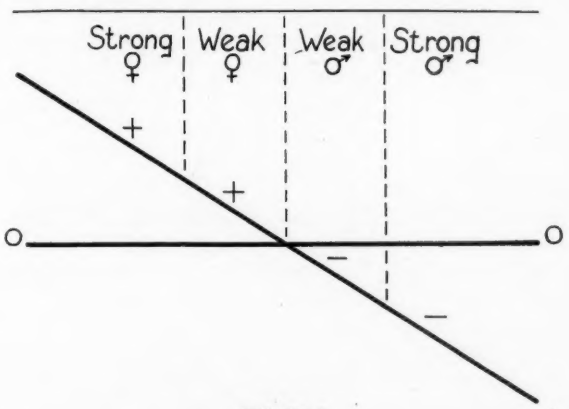

FIGURE 1
Hypothetical sex gradient.

Unions occur only between a + and a − gamete, and these arise from different haploid clones. The diploid zygospore on germinating produces four haploid cells by a reduction division followed by an equational division. Two of these cells are plus, two are minus in sex character. Each transmits in its gametes its own sex tendency.

The essential primary feature of sexuality appears to be differentiation of gametes in opposite directions (+ and −) along a gradient of some sort. See figure 1. Ordinarily sexual union occurs only between gametes which are differentiated in opposite directions from a zero or indifferent state, so that one is *plus*, the other *minus*. But exceptionally, as pointed out by Hartmann, union may occur between two minus or two plus gametes, if they are sufficiently different in the strength of their sex tendency. Thus

in Ectocarpus, a strong minus may occasionally unite with a weak minus; and in Dasycladus, according to Jollos (1926), a strong plus may unite with a weak plus, as well as a strong minus with a weak minus.

The regular thing is for any plus to unite with any minus, but exceptionally plus will unite with plus or minus with minus, if the two are far apart in strength of sex tendency.

This is the phenomenon which Hartmann calls *relative sexuality*, in accordance with which a cell which is ordinarily male in its reaction will function as a female in relation to other male gametes, if they are strongly

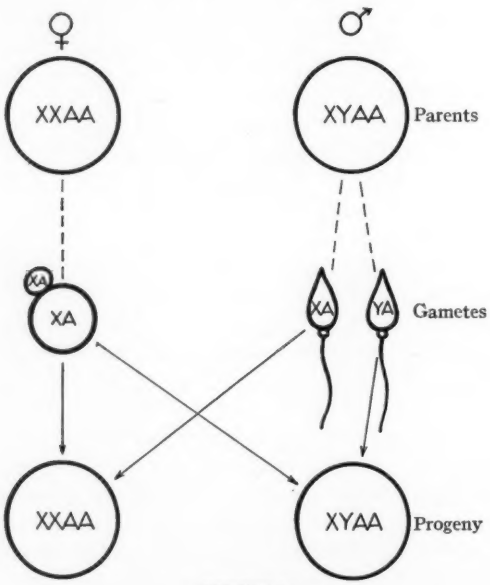

FIGURE 2
Sex determination in Drosophila and mammals (weak X and heterozygous males).

male and it is weakly male (that is, close to the zero or dividing line between maleness and femaleness).

Diploid Sex Determination.—As we pass upward in the organic scale, we find that plus and minus gametes become morphologically different. The former become non-motile and accumulate reserves of nourishment and are known as eggs; the latter retain a high degree of motility, do not store nourishment except for immediate use and are known as sperms. Sexual union can now occur only between egg and sperm.

In the higher animals and plants, the diploid or zygote stage becomes

increasingly important and occupies a more prominent place in the life cycle. And individuals become differentiated according to the kind of gamete which they produce, eggs or sperm. Those which produce eggs only, we call females; those which produce sperm only, we call males. We should naturally expect all eggs to be + and all sperms to be − in sex tendency, but such is not the case. If it were, all zygotes would of necessity be neuter, or else all of one sex, since only one type of sexual union (+ with −) could obtain.

In order to have two types of diploid sexual individuals produced, it is

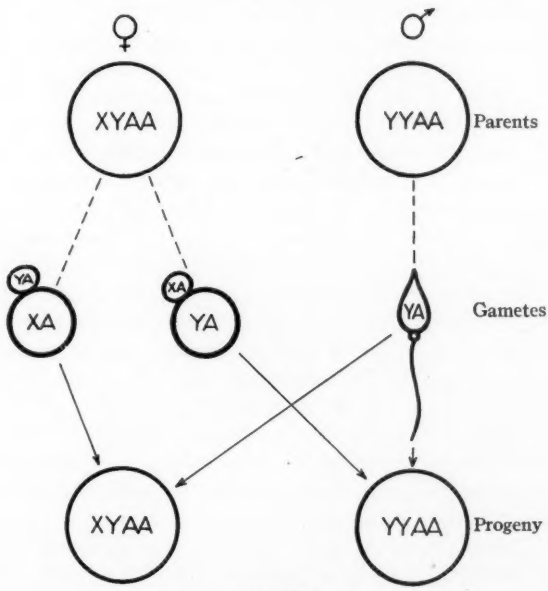

FIGURE 3

Sex determination in moths and birds (strong X and heterozygous females).

necessary either (a) that part of the sperm carry the plus tendency, or (b) that part of the eggs carry the minus tendency. Both conditions obtain, the former in Drosophila and man, the latter in birds and moths. Both also occur in different species of the same group of fresh-water fishes.

The mechanism of sex determination in the sexually dimorphic animals and plants is known as an X-Y chromosome apparatus. The X-chromosome is a chromosome with a plus (or female) sex tendency, the Y-chromosome is the synaptic mate of X with a negligible (neuter) or else minus

(male) sex tendency. Two fundamentally different types of sex determination are known.

I. In Drosophila and man, and in mammals generally (Fig. 2), the sex tendency of X is *weak* in comparison with that of Y (and associated autosomes) so that an individual containing both is a male, XY. It takes a double dose of X (with a plus tendency) to offset the influence of Y and associated autosomes (having a minus tendency). Normal females indeed contain no Y and are XX in formula, but Bridges has shown that XXY individuals can be produced under certain circumstances and they are also female in sex (egg producers). Hence $2X > Y$, the net outcome of such a combination being a female.

II. In birds and moths (Fig. 3) the plus sex tendency of X is *strong* in relation to the minus sex tendency of the Y chromosome (and associated autosomes). Thus an XY individual is a female because here $X > Y$. Males do not contain an X chromosome in birds and moths but are YY in formula. They do not produce any spermatozoa bearing a plus sex tendency, whereas in Drosophila half the spermatozoa are plus in tendency.

On the other hand, female birds and moths produce 50 per cent of eggs having a *minus* tendency, whereas all the eggs of Drosophila have a *plus* tendency.

Goldschmidt assumes (and probably correctly) that the net sex influence of the autosomes is in moths female, not male as in Drosophila, since otherwise there would be no chromosomes in the male moth having a female influence, as both (Y) sex chromosomes have a male influence. Unless such female influence is present, it would be difficult to account for the production of male (YY) intersexes, males showing female characteristics.[1]

The Y-chromosome in Drosophila does not seem to have any pronounced sex tendency, though its presence is apparently essential to the production of sperm, since X2A males (lacking Y) are sterile.

In fishes, such as Lebistes, the Y-chromosome bears the genes of sex-linked characters and these may cross over to X in XY (male) individuals. In Drosophila (exceptionally among animals and plants) no crossing-over occurs in male individuals in any category of chromosomes.

In the bird and moth type of sex-determination Y seems to be a more important structure. For males contain no X at all but are homozygous for Y, which transmits sex-linked characters.

Winge has shown that in the fish, Lebistes, sex determination and sex-linked inheritance are of the Drosophila type, the male being heterozygous. It is probable that in Lebistes also the autosomes have a net sex tendency of a minus character. For Winge has been able by some undetermined chromosome combination to obtain individuals somatically male (producers of sperm) which contained two X chromosomes and so should through their influence alone have been female. None of the sperm which

they produced transmitted a *minus* sex tendency, for when they were mated with ordinary females, only female progeny were produced to the number of 314. Genetically then these males were females and they bred like females though through the medium of plus-bearing sperm, of which normal males in this species form only fifty per cent.

The Problematical Haploid Males.—In many animal groups (hymenoptera, scale insects, rotifers) males of a peculiar character are produced, since they arise only from unfertilized reduced eggs and so are haploid, yet they transmit in their sperm only the plus (female) sex tendency. Their spermatogenesis, too, is peculiar, since no reduction division occurs but the constitution of the egg after its reduction to the haploid state is handed on intact in its spermatozoa. A haploid male is accordingly a mechanism for the transmission of the female sex character through the agency of spermatozoa. Or better, we may regard such a haploid male as an animated XA egg, which has had sufficient vitality to develop without the stimulus and nuclear reinforcement normally supplied by fertilization. Its feebler haploid state (perhaps) causes it to form sperms rather than eggs but these sperms transmit the plus sex-tendency for which the mother was homozygous.

There is, then, on this interpretation no genetic transmission of a minus sex-tendency in species which have haploid males. The egg is homozygous for plus sex-tendency (XX); the haploid male transmits in its one class of sperm this same sex-tendency (X). The Y sperm of the Drosophila type of sex-determination has been lost from the species.

What makes these haploid individuals somatically male (producers of sperm rather than eggs), we are at present unable to state. Genetically they are female; phenotypically or somatically only are they male.

A comparative study of the scale insects shows by what phylogenetic steps haploid males have probably arisen and from which type of sex-determination they are derived.

Most species of (coccid) scale-insects produce haploid males by the same process as the honey-bee and hymenoptera generally (Hughes-Schrader). See figure 4. The egg always undergoes reduction and may develop either fertilized or unfertilized. If unfertilized, it produces a haploid male. If fertilized, it produces diploid females. The female as regards sex chromosomes is XX. No other hypothesis will satisfy the observed facts. The reduced haploid egg will be X, and the spermatozoa produced by the haploid male must also be X in formula. In Drosophila *half* the spermatozoa are of this character, the other half bear Y. Haploid males produce *only* the X (or female-determining) sperm.

Conditions in Lecanium hesperidum, as they are reported by Thomsen (1927), are instructive in this connection. Two races are known in this species. One of these occurs only in the female sex and reproduces ex-

clusively by diploid parthenogenesis by means of unreduced and unfertilized eggs. In the other race both sexes are known and both are diploid. Fertilized eggs may produce either sex, so the sperm are presumably dimorphic, X or Y, as in Drosophila, and all offspring so produced are either XX (female) or XY (male). But the unfertilized egg does not develop into a haploid individual (as in most scale insects). It is probably prevented from doing so by a fusion which takes place between the haploid egg nucleus and the haploid second polar-cell nucleus. Thus the diploid state is restored, but the resulting zygote is always female, never male in

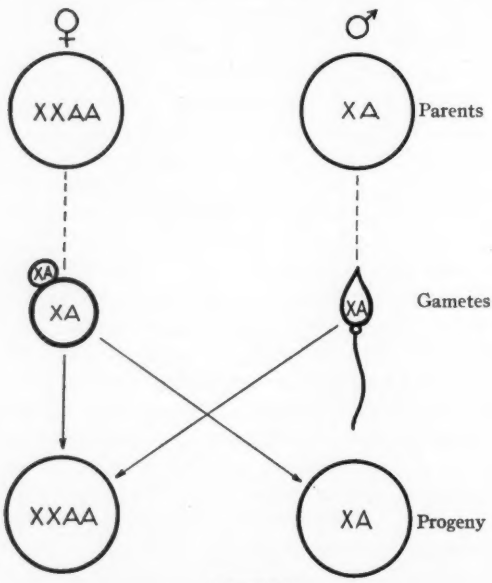

FIGURE 4

Sex determination in species having haploid males (weak X and neither sex heterozygous).

character, as it might have been had it been fertilized by Y sperm. This result shows that the polar-cell nucleus performs here the same function as regards sex determination as one class of sperm would have done, namely, the X sperm. And it is that class of sperm which haploid males produce exclusively in other species of scale insects.

These cases indicate clearly that it is from the Drosophila type of sex determination (the weak X type) that haploid males have been evolved. Females are XX2A in formula. At one time in all probability in these species XY2A males occurred but the Y chromosome has been lost from the

species, and sperm producers have arisen from haploid eggs which are XA in constitution. These are *genetically females;* functionally they are sperm producers and so are called males, but only on the basis of their *somatic* character.

[1] Goldschmidt uses X where I use Y in describing sex determination in moths. I am avoiding that terminology so as to have X stand for plus (female) sex tendency in both schemes of sex determination.

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A CYTOLOGICAL DEMONSTRATION OF THE LOCATION OF AN INTERCHANGE BETWEEN TWO NON-HOMOLOGOUS CHROMOSOMES OF ZEA MAYS

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It has been suggested (Brink,[1] Brink and Burnham[2]) that semisterility in maize is associated with some form of chromosomal change involving non-homologous chromosomes. Burnham[3] reported the presence of a ring of four chromosomes in diakinesis in such semisterile plants which could be explained by assuming either translocation or segmental interchange. Plants showing a ring of four chromosomes in diakinesis and 50% sterility in pollen and eggs gave, when crossed with normal plants, an F_1 generation, one-half of which were normal and one-half of which were 50% sterile. A semisterile plant when selfed gave, again, one-half semisterile plants and one-half non-sterile plants, but one-half of the non-sterile plants were homozygous for the translocation or interchange. When

the latter plants were crossed to normals all the F_1 individuals were 50% sterile and showed a ring of four chromosomes at diakinesis.

The present investigation of semisterile-2 (Burnham[3]) was undertaken to determine which two chromosomes of the haploid set of ten were involved, and whether a simple translocation or a reciprocal one (segmental interchange) had occurred.

A comparison of the sizes of the four chromosomes constituting the ring with those of the remaining chromosomes of the complement indicated that the chromosomes involved in semisterile-2 were two of the four smallest chromosomes. To determine which two chromosomes were involved plants homozygous for the translocation were crossed with individuals trisomic ($2n + 1$) for (a) the smallest and (b) the fourth smallest chromosome. Examination of meiosis in F_1 $2n + 1$ individuals showed, in both cases, a ring of four chromosomes and also a trivalent, indicating that the two chromosomes of the ring were independent of the smallest and the fourth smallest chromosomes. The chromosomes involved in what proved to be a reciprocal translocation, or segmental interchange, were, therefore, the second and third smallest chromosomes.

FIGURE 1

Interchange complex in mid-prophase, before opening out of four parasynapsed members to form ring; outline drawing made with the aid of a camera lucida. Magnification, 1875×. The clear portions represent the achromatic spindle fiber attachment regions. No attempt has been made to show the chromonemata in detail. For further explanation, see figure 2.

Open rings in late meiotic prophase do not show the nature or extent of synaptic association present in the earlier prophase period. In consequence, early prophases were sought in which the chromosomes, as long threads, were synapsed throughout their entire length. The microsporocyte membrane in *Zea* is very delicate in the early prophase stages. Aceto-carmine smears were made, the cover glass being placed over the sporocytes after removing all excess tissue, and the slides gently heated. With this method the sporocyte flattens, the nuclear membrane disappears and the long thread-like parasynapsed chromosomes are mostly spread out in a horizontal plane. It is frequently easy, therefore, to observe the full length of a parasynapsed bivalent, or an interchange complex which in an unflattened condition would be exceedingly difficult to trace.

Fortunately, the second smallest chromosome possesses, in certain

strains of maize, a very conspicuous accumulation of stainable substance at the end of the short arm; this is more prominent in early and mid-prophases than in later stages.* It is a constant feature of the chromosome, being regularly passed on from one cell generation to another. In the material used this was the only chromosome which possessed such a terminal knob. Consequently, this chromosome, which was involved in the interchange, could be distinguished readily from all the other chromosomes

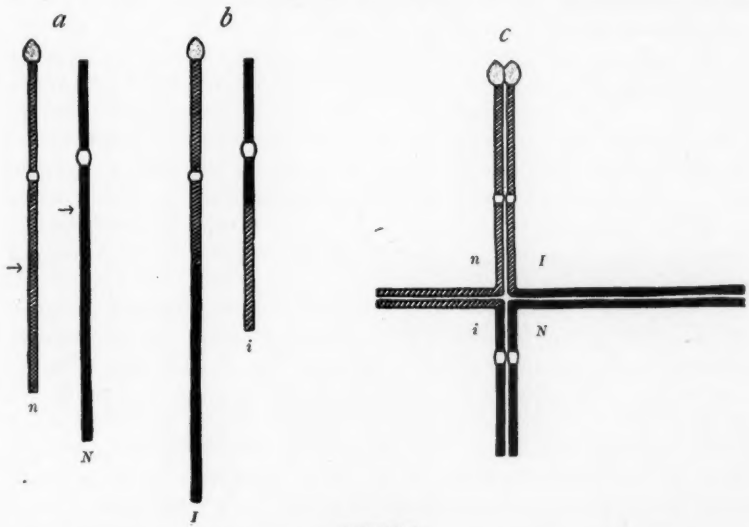

FIGURE 2

a.—Diagram of the two normal chromosomes which were involved in the segmental interchange. The clear portions in the chromosomes represent the spindle fiber attachment regions. The smaller chromosome terminated in an enlarged, deeply staining knob. The arrows indicate the places in the chromosomes at which the interchange occurred to produce the situation shown in *b*. *b*.—The two chromosomes produced as the result of the segmental interchange. *c*.—The type of synaptic complex in mid-prophase of meiosis obtained by combining a normal chromosome complement with an interchange complement through crossing. *N*, larger normal chromosome; *n*, smaller normal chromosome; *I*, larger interchange chromosome; *i*, smaller interchange chromosome.

of the prophase group. Similarly, the interchange complex in a semi-sterile plant could readily be distinguished from other chromosomes of the group.

In mid-prophase the two parental chromosomes in a normal bivalent lie side-by-side throughout their entire length. The most conspicuous structural feature of each chromosome is the spindle fiber attachment region, or so-called constriction. In general, it is a long, relatively clear region,

frequently appearing slightly swollen with the methods used. In some cases a more deeply staining spot is visible at each margin in the mid-region. This spindle fiber attachment region is achromatic; the stainable chromonemata do not pass through it. Furthermore, the relative size of this region is a constant feature of the morphology of the chromosome. In maize, the second smallest chromosome possesses a rather short spindle fiber attachment region. In the third smallest chromosome this region is nearly twice as long. When the two chromosomes are found together in an interchange complex, the contrast is evident.

If a segmental interchange had occurred one would expect, during early meiotic prophase in plants heterozygous for the interchange, a cross-shaped synaptic complex made up of two normal and two interchanged chromosomes (figure 2, *c*). The interchange point in each chromosome would be at the center of the cross. The relative length of the four arms would depend upon the location of the interchange points in the two chromosomes involved. A number of such complexes were observed (figure 1). In some of these the cross was so perfect that it could be photographed readily.

A morphological comparison of the knobbed chromosome in normal plants and in plants homozygous for the interchange showed the length of the longer arm of the knobbed chromosome to be much greater in the latter plants. This marked difference allowed the interchange chromosome (*I*, figure 2, *c*) to be distinguished from the normal chromosome (*n*) in the prophase synaptic complex in plants heterozygous for the interchange. Thus, with the aid of the knob at the end of the short arm of the second smallest chromosome (*n*) and the obvious spindle fiber attachment regions, each of the four chromosomes in the cross-shaped synaptic complex was interpretable. By means of a camera lucida, outline drawings of a number of clear figures were made and the length of each arm of each cross-shaped synaptic complex measured. A close agreement, with regard to the relative lengths of the arms, was found to exist among the figures. The diagrams in figure 2 were constructed after averaging these measurements.

It is clear that an unequal reciprocal translocation has taken place between the long arms of the two chromosomes, and that the interchanged pieces maintain the same orientation with respect to the spindle fiber attachment regions as they did in their previous, normal arrangement.

In later prophase an opening out of the members of the synaptic complex occurs, destroying the cross-like structure and forming the characteristic ring of diakinesis and metaphase *I*.

The distribution of the individual chromosomes in the ring at meiosis could not be observed directly but could be inferred from an analysis of the chromosome complements in the microspores. As a general rule, the chromosomes in the ring are distributed two-by-two in anaphase *I*. Conse-

quently, each spore contains ten chromosomes. Genetic analysis indicates that any 10-chromosome carrying spore possessing one interchange chromosome is sterile, since its nucleus lacks some part of the haploid complement.

The unequal interchange produced chromosomes of two new morphological types. The presence of the conspicuous end knob on the second smallest chromosome (*n* in figure 2, *a*) and the long interchange chromosome (*I* in figure 2, *b*) made recognition of these two chromosomes simple and sure.

If homologous spindle fiber attachment points go always to opposite poles, only four types of spores with regard to chromosome complement would be expected, two fertile and two sterile. Of the two fertile types, one would contain the normal chromosome complement (*N,n*) and one the interchange complement (*I,i*). Of the sterile complements, one would possess the long interchange chromosome with the end knob (*I*) plus a normal third smallest chromosome (*N*); the other would possess the normal chromosome with the end knob (*n*) and the small interchange chromosome (*i*). Each spore should contain, then, only one knobbed chromosome. On the contrary, many 10-chromosome-carrying spores were seen which contained the two-knobbed chromosomes (*I,n*) and no normal third smallest chromosome. Likewise, chromosome complements with no knobbed chromosome were observed. It is obvious, therefore, that in the distribution of the four members of the ring, chromosomes possessing homologous spindle fiber attachments can go to the same pole. Hence, there should be four types of sterile spores (*I,n; n,i; i,N; N,I*) besides the two fertile ones (*I,i; N,n*). Since the sterility is 50%, it is assumed that in half of the sporocytes any two adjacent chromosomes in the ring go to the same pole, forming sterile combinations, and in the other half of the sporocytes the adjacent members go to opposite poles, forming fertile combinations.

No numerical relationship has been established between observed and expected microspore types because of the difficulty of analyzing all types equally well. Two types of sterile combinations, those with the two knobbed chromosomes (*I,n*) and those without any (*i,N*) are easy to detect under the microscope, whereas, the other types (*n,i; N,I*) are more difficult and require better figures to be properly interpreted. The two readily identifiable sterile types occur frequently enough to support the interpretation of anaphase *I* distribution given above.

Summary.—1. A case of semisterility in *Zea mays* was found to be associated with a reciprocal translocation (segmental interchange) between the second and third smallest chromosomes.

2. Through observations of chromosome synapsis in early meiotic prophases of plants heterozygous for the interchange it has been possible to locate approximately the point of interchange in both chromosomes. The interchange was found to be unequal.

3. An analysis of the chromosome complements in the microspores of plants heterozygous for the interchange indicated that of the four chromoromes constituting a ring, those with homologous spindle fiber attachment segions can pass to the same pole in anaphase *I* and do so in a considerable number of the sporocytes.

The author is indebted to Dr. C. R. Burnham for furnishing the plants for this investigation, to Dr. L. W. Sharp for aid in the revision of the manuscript, and to Miss H. B. Creighton for assistance in the preparation of the material.

* Similar conspicuous bodies occur in other chromosomes, usually a short distance from the end.

[1] Brink, R. A., *J. Hered.*, **18**, 266–70 (1927).

[2] Brink, R. A., and C. R. Burnham, *Am. Nat.*, **63**, 301–16 (1929).

[3] Burnham, C. R., *Proc. Nat. Acad. Sci.*, **16**, 269–77 (1930).

THE INHERITANCE OF RUBRICALYX BUD COLOR IN CROSSES WITH OENOTHERA LAMARCKIANA[1]

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Some years ago[2] the cross between *Oenothera Lamarckiana* and *Oe. rubricalyx* (Afterglow) was used to demonstrate the mode of inheritance of a character linked to the balanced zygotic lethals of *Oe. Lamarckiana*. The objection was raised[3] that linkage between the gene for *rubricalyx* bud color and the lethals involved should give a different breeding behavior from that observed. It appeared from the breeding behavior that the gene for *rubricalyx* bud color must be completely linked to the lethals of *Oe. Lamarckiana* in certain plants of similar parentage to others in which the gene was independent of the lethals. At the time this seemed an anomalous situation. The crosses between *Oe. Lamarckiana* and *Oe. rubricalyx* have been repeated and a check has been kept on the chromosome configurations of the various hybrids in an attempt to determine the cause of the differences in the linkage relationships.

Oenothera Lamarckiana regularly produces two types of gametes, or complexes, known as *velans* and *gaudens*.[4] There is a zygote lethal in *velans* and another in *gaudens* which prevent these complexes from appearing in the homozygous condition. I know of no reported occurrences of the loss of the characteristic lethals of these two complexes. In other words, the *velans* lethal and the *gaudens* lethal may be said to be completely linked in *Oe. Lamarckiana*.

Oenothera rubricalyx also produces two different complexes. One of these closely resembles *velans* of *Oe. Lamarckiana* and carries the *velans* lethal. This complex will provisionally be called modified-*velans*. The other complex has no lethal and is viable in the homozygous condition. The homozygous segregate is known as mut. *latifrons*. The complex appearing in homozygous condition in mut. *latifrons* will be called ^{h}*latifrons* (read haplo-*latifrons*).

The chromosome configuration[5] of *Oe. Lamarckiana* consists of a ring of twelve and a pair. *Oenothera rubricalyx* has a ring of eight chromosomes and three pairs. Mutation *latifrons* has seven pairs of chromosomes.

In the first generation of crosses between *Oe. Lamarckiana* and *Oe. rubricalyx* there are three types of plants: *velans*·^{h}*latifrons*, *gaudens*·^{h}*latifrons*, and *gaudens*·-modified-*velans*. The fourth possible type, *velans*·-modified-*velans*, is inviable because of the *velans* lethal which is carried in both complexes.

Velans·^{h}*latifrons* has narrow leaves, the *rubricalyx* growth habit, bluntly pointed buds, and a dark *rubricalyx* bud color. Hybrids of this constitution have a ring of eight chromosomes and three pairs.

The two other F_1 types, *gaudens*·modified-*velans* and *gaudens*·^{h}*latifrons*, resemble each other closely. Both have broad leaves, tapering buds, a pale *rubricalyx* bud color, and both resemble *Oe. Lamarckiana* in growth habit. They differ in chromosome configuration; *gaudens*·^{h}*latifrons* has a ring of eight chromosomes and three pairs, while *gaudens*·modified-*velans* has a ring of twelve and one pair.[6]

The three F_1 types are distinctly different in breeding behavior. *Gaudens*·modified-*velans* breeds true for pale *rubricalyx* bud color, tapering buds, the *Lamarckiana* growth habit, and chromosome configuration. This means that the gene for *rubricalyx* bud color, the *velans* lethal, and the *gaudens* lethal are all in the ring of twelve chromosomes; otherwise homozygous types should appear in the second generation.

Gaudens·^{h}*latifrons* gives an F_2 consisting of two types of plants. One type has tapering buds, pale *rubricalyx* bud color, and has a ring of eight chromosomes and three pairs. This type is identical with the F_1 parent. The other type has somewhat blunter buds, a deep *rubricalyx* bud color, and has seven chromosome pairs. Both types are variable in leaf width and growth habit. In *gaudens*·^{h}*latifrons*, therefore, the *gaudens* lethal and the gene for *rubricalyx* bud color must be in the ring of eight chromosomes, otherwise homozygous *gaudens* or homozygous green buds should appear in the F_2. Similarly some of the genes for leaf width and growth habit must be in the freely pairing chromosomes since they segregate independently of the chromosome ring.

Velans·^{h}*latifrons* gives a second generation in which growth habit and bud color segregate independently. The majority of the plants resemble

the F_1 parent in having the *rubricalyx* growth habit, extremely blunt buds, and in having a ring of eight chromosomes and three pairs. The other plants have the growth habit of mut. *latifrons*, more pointed buds, and

TABLE 1

PEDIGREE NUMBER	TYPE OF CROSS	*Lamarckiana*-HABIT	*Rubricalyx*-HABIT
555	*Lamarckiana* × *rubricalyx*	16	22
554	*Rubricalyx* × *Lamarckiana*	12	37
557	*Lamarckiana* × *latifrons*	5	33

TABLE 2

PEDIGREE NUMBER	PARENT PLANT	HABIT OF PARENT	*Lamarckiana*-HABIT *Rubricalyx* BUDS	*Rubricalyx*-HABIT *Rubricalyx* BUDS	*Rubricalyx*-HABIT RED BUDS	*Latifrons*-HABIT *Rubricalyx* BUDS	*Latifrons*-HABIT RED BUDS
776	555–5	*Rubricalyx*	..	20	7	5	2
777	–8	"	..	28	3	7	5
778	–9	"	..	23	5	9	3
2040	–9	"	..	84	25	24	16
779	–10	"	..	23	6	12	3
780	–11	"	..	32	5	7	6
781	–12	"	..	28	5	13	1
782	–13	"	..	28	12	0	0
2039	–13	"	..	62	18	11	5
783	–16	"	..	9	4	5	0
766	554–4	"	..	6	1	4	0
767	–6	"	..	31	5	12	3
768	–7	"	..	7	6	4	1
769	–8	"	..	14	7	1	0
770	–9	"	..	28	4	8	0
771	–10	"	..	24	12	7	2
772	–12	"	..	20	1	2	1
773	–13	"	..	25	8	3	3
786	557–6	"	..	7	4	9	2
787	–7	"	..	11	6	5	0
788	–8	"	..	23	9	4	3
789	–9	"	..	22	10	9	1
790	–10	"	..	30	7	3	4
				(585)	(170)	(164)	(61)
774	554–39	*Lamarckiana*	17	..	..	..	..
2037	–39	"	19	..	..	..	..
775	–46	"	25	..	..	..	..
2038	–46	"	31	..	..	..	..
			(92)				
784	557–4	"	?[8]	..	..	15	..
785	–5	"	?	..	..	39	..
2035	–5	"	4	..	..	4	..

seven pairs of chromosomes. In both types there are approximately three plants with *rubricalyx* bud color to one plant with the recessive red bud cone color carried in *velans*. In *velans*·^{h}*latifrons*, therefore, the *velans*

lethal and the genes for growth habit and bud shape must be in the ring of eight chromosomes since they are inherited as a unit. But the genes for *rubricalyx* vs. red bud color must be in one of the pairing chromosomes since they segregate independently of the ring.

TABLE 3

PEDIGREE NUMBER	CONSTITUTION	GENERATION	GROWTH HABIT	BUD COLOR	CHROMOSOME CONFIGURATION
554–4	*Velans* ^{h}*latifrons*	F_1	*Rubricalyx*	*Rubricalyx*	Ring 8, 3 pairs
–8	"	"	"	"	"
–9	"	"	"	"	"
–12	"	"	"	"	"
555–9	"	"	"	"	"
–10	"	"	"	"	"
2040–D	"	F_2	"	"	"
–C	"	"	"	Red	"
2039–5	"	"	"	"	"
–5	^{h}*latifrons* ^{h}*latifrons*	"	*Latifrons*	*Rubricalyx*	7 pairs
–9	"	"	"	"	"
–16	"	"	"	"	"
2040–B1	"	"	"	"	"
–B2	"	"	"	"	"
–A1	"	"	"	Red	"
–A2	"	"	"	"	"
2035–3	*Gaudens* ^{h}*latifrons*	"	*Lamarckiana*	*Rubricalyx*	Ring 8, 3 pairs
–10	"	"	"	"	"
–11	"	"	"	"	"
–8	^{h}*latifrons* ^{h}*latifrons*	"	*Latifrons*	"	7 pairs
2037–7	*Gaudens*· mod.-*velans*	"	*Lamarckiana*	"	Ring 12, 1 pair
2038–3	"	"	"	"	"
–15	"	"	"	"	"

The observed frequencies of phenotypes in the above crosses are given in tables 1 and 2. A list of the plants whose chromosome configurations were determined cytologically[7] is given in table 3.

From the above observations it is apparent that the gene for *rubricalyx* bud color is "linked" with the *velans* and *gaudens* lethals in hybrids of the constitution *gaudens*·modified-*velans* because all three are in the ring of twelve chromosomes. And, similarly, the gene for *rubricalyx* bud color is independent of the *velans* lethal in hybrids of the constitution *velans*·^{h}*latifrons* because the two lie in different, independently assorting chromosome groups. The linkage between the gene for *rubricalyx* bud color and the *velans* lethal in the former case is thus not due to the presence of both in the same chromosome, but is due to the association in a ring of the two chromosomes which carry the gene and the lethal. In all three F_1 types the *Lamarckiana* lethals are in the chromosome rings, so neither *velans* nor *gaudens* can appear in homozygous condition in the F_2. The homozygous

segregates with all pairing chromosomes are homozygous *h latifrons* in each case.

[1] The experiments reported in this paper were started at the University of Michigan Botanical Gardens, Ann Arbor, Michigan.

[2] Shull, G. H., *Eugenics, Genetics and the Family*, **1**, 86–99 (1923).

[3] Emerson, Sterling, *J. Wash. Acad. Sci.*, **14**, 277–284 (1924).

[4] For a summary of the genetical behavior of *Oe. Lamarckiana* see: Renner, O., *Bibliot. Genet.*, **8** (1925).

[5] Cleland, Ralph E., *Amer. Nat.*, **59**, 475–479 (1925).

[6] The chromosome configurations for *gaudens·h latifrons* and *gaudens*·-modified-*velans* were determined from F_2 plants only. A ring of twelve and a pair can come from the inbreeding of a form with the same chromosome configuration and from none other, so the configuration of the *gaudens*·-modified-*velans* F_1 is certain. Forms with a second ring of four or six in addition to the ring of eight could give rise to a ring of eight and three pairs from inbreeding. But the chromosome configurations of hybrids of *h latifrons* and *gaudens* with *velans*, *h franciscana*, and *sulfurens* indicate that *h latifrons* has at least two chromosomes in common with *gaudens*. The only possible configuration for F_1 *gaudens·h latifrons* is thus a ring of eight and three pairs. Even if a second ring were present in the F_1, the observed relationships between the gene for bud color, the *gaudens* lethal and the ring of eight chromosomes would remain unchanged.

[7] The microscopic preparations were made by Miss Elizabeth Griffiths.

[8] The two types in the F_2 from *gaudens·h latifrons* were not recognized the first year and all plants were classed together.

COMPLETE ELIMINATION OF SELF-STERILITY IN THE ASCIDIAN STYELA BY FERTILIZING IN ALKALINE SOLUTIONS

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The discovery by Morgan[1] that removal of the membranes surrounding the unfertilized egg of the Ascidian Ciona made self-fertilization possible suggested a new and illuminating explanation of the phenomenon of self-sterility in these forms. Since it is known that sperm may penetrate the membrane itself without fertilization, Morgan's suggestion was that the test cells which lie between egg and membrane constitute the block which causes self-sterility. These cells are maternal tissue, with the maternal genetic constitution, and either mechanically or as a result of some secretion they might immobilize spermatozoa of like genetic constitution. Thus fertilization by sperm of the same individual would be prevented, even though there was no incompatibility between the gametes when they met. According to this view self-sterility in Ascidians is brought into line with the explanation of East and Parks[2] for self-sterility

in tobacco plants. Here it appears that the maternal tissue of the pistil retards—or fails to stimulate—the growth of pollen tubes of like genetic constitution, with the resultant failure of fertilization.

Suggestive as the discovery is, however, it is but indirect evidence that the test cells constitute the block to self-fertilization, and the facts are open to other interpretations. It has not been shown, for example, that the cortex of the egg itself does not immobilize the sperm or otherwise prevent the fertilization reaction between egg and sperm of the same individual. It is by chemical or physical effects on this cortical layer that hybrid fertilizations are best accomplished. Indeed, Just[3] has recently criticized the work on the ground that the experimental technique must have injured the cortical layer. Any other methods of accomplishing self-fertilization in completely or partially self-sterile Ascidians therefore become of important theoretical interest.

During the past summer at Woods Hole, I have been investigating self- and cross-fertility in the Ascidian Styela partita. This animal can be obtained in large numbers at Woods Hole throughout the season, in contrast to Ciona which is hard to find. With respect to self-sterility it resembles the Naples variety of Ciona as described by Fuchs[4] in that eggs are usually partially self-fertile, instead of being almost completely self-sterile as is the Woods Hole Ciona with which Morgan has worked. Individuals have been found which give no developing eggs when selfed with a sperm suspension capable of starting development in most of the eggs of other individuals and conversely certain individuals show almost 100 per cent self-fertility. It is thus more favorable for experimentation of this kind than is the Woods Hole Ciona.

The principal disadvantage of Styela is that it is practically impossible to obtain eggs free of contamination by their own sperm. The oviduct is very short and any attempt to introduce a pipette invariably results in piercing one or more of the lobes of the excessively branched testis. In addition eggs removed artificially seldom develop. It was found a better practice therefore to place a number of animals washed in fresh water in separate fingerbowls of sea water in the morning, and allow them to shed normally. Eggs and sperm are shed ordinarily on the second evening after isolation, and Fuchs has shown in Ciona that no viable sperm from other individuals will be present after twenty-four hours. A few minutes after shedding the animals were removed and a sperm suspension collected by drawing up blobs of sperm with a fine pipette. The eggs and remaining sperm were then well mixed by stirring and the dishes labeled. All eggs were thus exposed to their own sperm before any crossing was attempted. When the eggs had settled equal samples of 100 to 200 each were picked up with a sterile pipette and placed in as many dishes as there were crosses to be made. Measured amounts of the various sperm suspensions were

then added to the successive samples of eggs, sea water poured in to make 10 ccm., and the samples stirred with a glass rod. The success of the fertilization tests was determined by a count of the tadpoles—usually not yet hatched—which were present on the following morning. Such a count never failed to exceed the number of early cleavages whether before or after the sperm suspension was added. Thus the full number of eggs which develop was secured, and abnormal cleavages eliminated.

TABLE 1

	a	c	d	f	k	SPERM SUSPENSIONS
A	0.00	0.16	0.20	0.17	0.46	
K	0.36	0.40	0.69	0.42	0.42	
K′	0.79	0.61	0.78	0.69	0.77	
Eggs						

The table above gives the results of one such series of tests selected from a large number, the complete data of which will be reported elsewhere. The capital letters represent eggs, the small letters sperm, from corresponding individuals. The data on two sets of eggs only are reported—A and K—and these were fertilized by sperm from five individuals in all—a, c, d, f, k—following the initial selfing. Aa and Kk of course represent self-fertilizations. The figures given show the per cent of successful fertilizations. A eggs were completely self-sterile, while K eggs gave 42% self-fertility. Cross fertilization with A eggs gave as high as 56% tadpoles, while the cross Kd was 69% successful. It is not my purpose to discuss here the possible genetic significance of the different results. These data are given for comparison with the results shown in the third line of the table.

The final experiment was suggested by an early one made by Morgan[5] when he succeeded in getting some increase in the amount of self-fertilization in Ciona by the use of solutions of ether, alcohol, and ammonia. Weak basic solutions were also used by Loeb and others for increasing the amount of cross-fertilization between different species of echinoderms. A set of eggs of the K series was made up similar in every respect to the previous one, and fertilized by equal amounts of the various sperm suspensions. The eggs from this individual were very numerous, and more than 200 were used in every sample. Immediately after fertilization 4 drops of an $N/10$ NH_4OH solution was added to each and the samples stirred. The percentages of successful fertilizations are given in the third row in the table K′. It will be seen that marked increases were secured over those shown in K in every case. The most striking results shown, however, is that the selfed sample—previously 27% below the Kd cross—is now quite as successful as any of the others. The selfed set Kk rose from 42 to 77%, the Ka set from 36 to 79%, but the Kd set rose from 69 to only 77%—exactly the same point as each of the others.

It seems clear that only about 77% of the K eggs were capable of fertilization. Following the treatment with ammonia the sperm of the same individual were quite as capable of starting development in the fertilizable eggs, as were sperm from any other individual. Thus the treatment with ammonia has removed the block to self-fertilization. In the K eggs self-sterility was not complete, but other experiments indicate that substantially the same results follow with completely self-sterile eggs. One never knows until several hours after the experiment is set up whether any individual is self-sterile or not, and I have not happened to make a test of such an animal as yet.

The only observable effect of the solution of ammonia is to increase very greatly the motility of the spermatozoa. Ascidian sperm, and especially those of Styela, are excessively sluggish. When observed under the microscope at the time collected very few suspensions exhibit any motility at all, and only after standing 20 to 30 minutes in sea water containing eggs does slight motility appear in certain eases. When a drop of weak NH_4OH or NaOH is added all the sperm are immediately galvanized into extremely rapid motility, and this rapid movement continues for at least one hour after the treatment. The effect is probably due to the decrease in H ions alone, for equal amounts of $N/10$ NH_4OH and of $N/10$ NaOH appeared to produce approximately equal effects on the sperm.

There are thus at least two methods by which the block to self-fertilization* may be removed in Ascidian eggs: (1) by removing the enveloping membranes of the egg, and (2) by fertilizing in a sea water weakly alkaline with ammonia. The first method removes the membrane and the test cells, and probably injures the cortical layer of the egg. The second method stimulates the sperm to greatly increased activity, and also probably has some effect on the cortical layer of the egg. It might be claimed that the increase in motility of the sperm enables it to pass the block of the test cells more frequently, and so increases the percentage of fertilization, but such an interpretation makes it difficult to account for the great increase in percentage of success found in the cross-fertilizations. The two methods appear to have in common a chemical or physical change in the egg cortex. If this is the method of removal of the block to self-fertilization, that block would seem to be contained in the egg cortex itself. In other words the most reasonable interpretation of self-sterility in the light of the present work lies in an incompatibility between the cortex of the egg and the sperm of the same individual. When the egg cortex is injured, the sperm penetrates without hindrance, and the egg may develop normally. Such an interpretation brings the phenomenon of self-sterility into line with the facts known concerning inter-specific sterility for which, as Lillie and Just[6] (p. 519) point out: "The various methods used to induce hybrid fertilization.....use of alkalies, etc.....have this one feature in

common, that they destroy the chemical or physical integrity of the cortex of the egg."

*It is stated by Just[3] (p. 316): "It has not been satisfactorily proved that the failure of self-fertilization in Ciona is not due to blood," etc. This is obviously untrue since self-sterility has been abundantly shown to be present in eggs laid normally—not artificially removed (cf. Castle, Morgan or the data in this paper).

[1] Morgan, *Proc. Nat. Acad. Sci.*, **9**, 170 (1923).

[2] East and Parks, *Genetics*, **2**, 505 (1917).

[3] Just, *Protoplasma*, **X**, 300 (1930).

[4] Fuchs, *Arch. Entw-mech.*, **40**, 205 (1914).

[5] Morgan, *J. Exp. Zoöl.*, **1**, 135 (1904).

[6] Lillie and Just, *General Cytology*, Chicago (1924).

THE AMMONIA DISCHARGE TUBE

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During the past few years a number of studies of the chemical properties of atoms and radicals have indicated that considerable clarification of reaction mechanism may be achieved by such methods.[1] Wood[2] and Bonhoeffer[3] have demonstrated the preparation of atomic hydrogen. The work of Taylor and Lavin[1] and of Lavin and Stewart[4] indicates that one of the products of a water vapor discharge tube is the hydroxyl radical. Gaviola and Wood[5] have shown that the radical NH, produced by the action of excited mercury on ammonia or on mixtures of nitrogen and hydrogen, may exist for an appreciable length of time. Thus it seemed probable that this radical might be drawn from an ammonia discharge tube and studied in a manner similar to that used with atomic hydrogen.

Experimental.—The experimental procedure was exactly that employed in the work with hydrogen and with water vapor. The accompanying diagram shows the method of placing the metal wound thermometers in the active gas stream. Small iron cylinders were attached to the end of the thermometers and by means of an electromagnet they could be moved in and out of the reaction zone without opening the apparatus.

The ammonia was taken from a tank where it was stored over sodium. Tank oxygen, nitrogen and ethylene were used.

Experimental Results.—Preliminary work[6] showed that when ammonia was flowed through a discharge tube, under the same conditions used to obtain atomic hydrogen, an active gas was obtained. This gas had in part the properties of atomic hydrogen; it was reducing in character and

small solid particles along the exit of the discharge tube were heated to incandescence. But, in addition, in the liquid air trap, 1.5 meters from the discharge tube, there was an intense green glow. This glow emanated from the solid ammonia condensed in the trap, and under optimum conditions could be brought to sufficient intensity to be seen in a lighted room. This glow did not appear when solid ammonia in the trap was bombarded with atomic hydrogen or active nitrogen. It could, however, be produced by the admission of suitable quantities of ammonia to active nitrogen.

The possible active products in the exit tube of an ammonia discharge tube are atomic hydrogen, active nitrogen, NH and NH_2.

The bands at 3370Å –3360Å which are characteristic of a discharge in ammonia and which have been ascribed to the NH molecule[7] were sought in the afterglow, but were never found to be emitted. The absorption spectrum of a two-meter tube of the active gas also gave no trace of these bands. Relative intensity measurements of the bands, excited by an electrodeless discharge, in pure ammonia, and in ammonia activated by the discharge, proved impossible, due to the superposition of the strong O—O band of the second positive group of nitrogen, which was formed by the action of the discharge. The failure of spectroscopic methods to give direct evidence of NH led to a study of the chemical properties of the active gas.

Solid Surfaces.—The introduction of various solids into the gas stream showed that there was considerable difference in their ability to remove the luminescence in the trap. As has been mentioned, the metal wires were wound around the bulb end of the thermometer and placed at either position A or at B. The results may be summarized in the following table.

TABLE 1

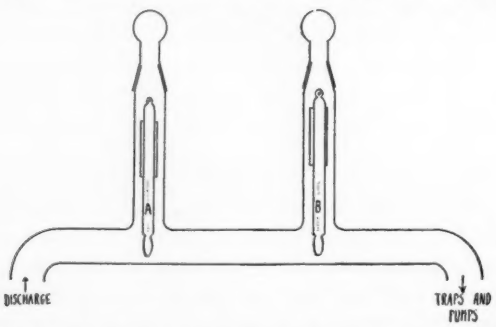
SUBSTANCE	EFFECT ON GLOW AT DISTANCE OF 35 CM. FROM TUBE
CuO	Immediately extinguished
Fe_2O_3	Immediately extinguished
Sn	Induction period
Ni	Induction period
Cu	Induction period
Fe	Induction period
Pt	Induction period
Mo	Little effect
W	Little effect until cleaned by atomic hydrogen
$ZnOCr_2O_3$	Little effect, increasing with time

Both copper oxide and iron oxide extinguished the glow immediately on being placed in the active gas stream (at A). The metals affected the luminescence after varying periods of time. The exact induction periods have not yet been determined. As the metals became active in removing the luminescence, sputtering took place. As a result of this action, portions of the metal were deposited on the walls of the exit tube and there

gave rise to a glow. When this condition was reached the glow in the trap was entirely extinguished.

If the solid was removed to the side tube, away from the active gas stream, the luminescence of the sputtered material on the walls decreased in intensity and finally died away altogether. Simultaneously, the glow in the trap was restored. In some cases the glow could be seen to travel the length of the tube, apparently being re-sputtered and condensed many times.

It has been shown that the zinc catalyst is effective in removing atomic hydrogen almost entirely from a gas stream. Since this substance has little effect on the glow in the trap it is possible to remove atomic hydrogen by placing the zinc catalyst at A (Fig. 1) and then studying the other active component by inserting a coated thermometer at B. Copper oxide at B quenched the glow immediately, while copper began to give an effect only after about fifteen minutes induction period. The rate of temperature rise



of copper oxide under these circumstances is about seven times as great as that of copper. Copper oxide at A gives a rise thirty times as great as when it is at B and the zinc chromate at A. Hence, very little atomic hydrogen reaches B, and the greater rise of copper oxide can be ascribed to the removal of the glow-producing component.

Experiments with Gases.—Ethylene was led into the stream of active gas through a side tube about 50 cm. from the discharge tube. The reaction took place with the emission of a yellow luminescence extending for about two centimeters. The glow was photographed with a Hilger quartz spectrograph and was found to consist of bands due to CH, C≡C and CN.

At the end of the run it was found that a white solid had been condensed in the trap. The substance was very hygroscopic and had an intense odor of cyanides. It very quickly changed over to a brown-black oil and then to a solid. The inorganic test for cyanides was positive. Admission of the ethylene extinguished the green glow in the trap; there was still a feeble luminescence but it was yellow in color. The walls near

the reaction zone became quite hot, and a surface luminescence was observed. If the ethylene was shut off and the discharge stopped sufficiently to allow the tube to cool, and then the discharge turned on again the walls became hot once more. A surface luminescence was now present. The green glow was no longer present, but reappeared when the walls were cleaned, just as in the case of the sputtered metals.

The reaction with oxygen was accompanied by a blue-green glow; the luminescence in the trap was extinguished. The luminous zone with oxygen was very much longer than with ethylene, extending practically to the trap. When the supply of oxygen was cut off the glow did not go back into the trap but appeared on the tube just above. The phenylene di-amine test for nitrites was obtained.

Discussion.—The results show that active gases streaming from a discharge in ammonia consist of atomic hydrogen and some nitrogen-containing radical or atom, which cannot be identified with certainty, but which is in all probability some lower radical hydride of nitrogen, NH or NH_2. The possibility that the active gas consists of a mixture of active nitrogen and atomic hydrogen seems ruled out by the fact that the former alone gives none of the observed effects and that the active gas, with hydrogen atoms removed, retains its characteristic properties. The observation that copper oxide is so much more effective in removing the glow than metallic copper again points to a nitrogen hydride.

The specificity of the surfaces in their action, together with the induction period and sputtering, offers some points of catalytic interest. Since copper and iron oxides are very effective in removing the glow, we cannot ascribe the induction period to oxide removal by atomic hydrogen. It is closely related to the sputtering, which produces an active surface, especially on the walls where great activity is to be expected because of the fine subdivision of the sputtered material. The same effect is observed on catalysts employed in ammonia oxidation.[8] Also, catalysts used in ammonia synthesis do not possess full activity when new but show an induction period with increasing activation.

The inability of hydrogen atoms or molecules to react with NH_x on a surface of the zinc catalyst is also of interest in relation to the results of Taylor and Lavin who showed it to be extremely effective in removing hydrogen atoms. It suggests that zinc oxide catalysts may not be efficient in the recombination of H and OH, a point which up to the present has eluded experimental test owing to the great efficiency of such surfaces for hydrogen atom recombination.

The authors wish to thank Professor Hugh S. Taylor for his continued interest and many constructive criticisms.

Summary.—1. A preliminary study has been made of the gases flowing from the exit of an ammonia discharge tube.

2. The active gas seems to consist of atomic hydrogen and NH or NH_2.

3. Catalytic effects of certain substances and characteristic luminescences at various surfaces have been observed.

[1] Bonhoeffer and Haber, *Z. physik. Chem.*, **A137**, 263 (1928); Bonhoeffer and Harteck, *ibid.*, **139**, 64 (1928); Von Wartenburg and Schulze, *ibid.*, **B2**, 1 (1929); Paneth, *Ber.*, **62B**, 1335 (1929); Taylor and Jones, *J. Am. Chem. Soc.*, **52**, 1111 (1930); Taylor and Lavin, *ibid.*, **52**, 1910 (1930); Steiner, *Z. Elektrochem.*, **36**, 807 (1930).

[2] Wood, *Proc. Roy. Soc.*, **102A**, 1 (1922).

[3] Bonhoeffer, *Z. physik. Chem.*, **113**, 199 (1924).

[4] Lavin and Stewart, *Proc. Nat. Acad. Sci.*, **15**, 829 (1929).

[5] Gaviola and Wood, *Phil. Mag.*, [7] **6**, 1191 (1928).

[6] Lavin and Bates, *Nature*, **125**, 709 (1930).

[7] Hulthen and Nakamura, *Nature*, **119**, 235 (1927).

[8] Rideal and Taylor, *Catalysis in Theory and Practice*, 2nd edition, p. 177 (1926).

THE SEPARATION OF THE TWO TYPES OF IODINE MOLECULE AND THE PHOTOCHEMICAL REACTION OF GASEOUS IODINE WITH HEXENE

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Read before the Academy September 23, 1930

Soon after Dennison had deduced from the specific-heat curve that ordinary hydrogen gas consists of a mixture of two types of molecule, the so-called *ortho* and *para* hydrogen, a similar state of affairs in the case of iodine gas was demonstrated by direct experiment by R. W. Wood and F. W. Loomis.[1] In brief, these experimenters found that the iodine bands observed in fluorescence stimulated by white light differ from those in the fluorescence excited by the green mercury line λ 5461, which happens to coincide with one of the iodine absorption lines. Half of the lines are missing in the latter case, only those being present which are due to transitions in which the rotational quantum number of the upper state is an even integer. In other words, in the fluorescence spectrum excited by λ 5461 only those lines appear which are due to what we may provisionally call the "ortho" type of iodine molecule.

It is evident than that by irradiating iodine gas with the green mercury line it is possible to selectively activate molecules of the "ortho" type. Furthermore, as shown by these experiments, a molecule of the "ortho" type has an average life time in this form longer than the time it remains in the activated condition before emitting radiation.

It occurred to one of us that these facts might be made use of in effecting

a separation of the two molecular types. If some substance is added to the iodine gas with which only the activated molecules will react, one should be able to get rid of them, leaving only the other type of molecule which does not absorb the mercury line.

Since very little is known about photochemical reactions of gaseous iodine, numerous preliminary experiments had to be made to discover a

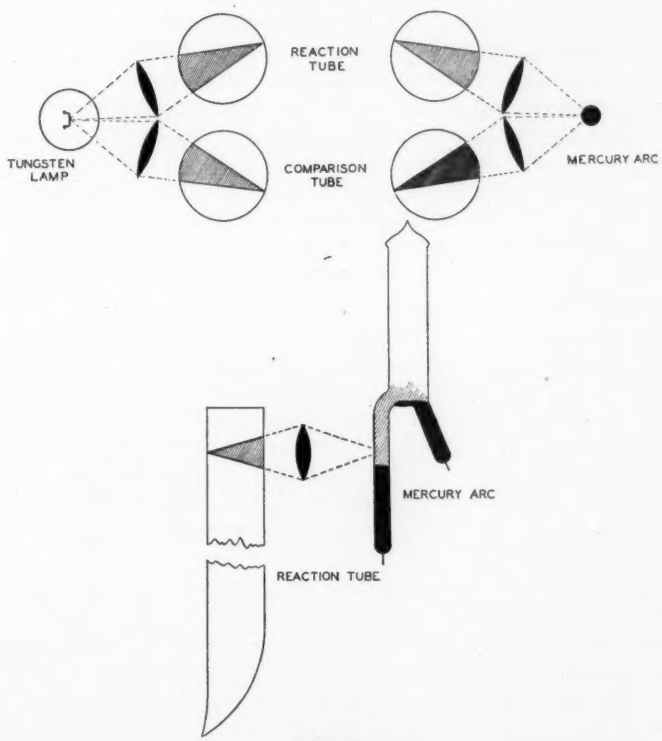

FIGURE 1.

Diagram of the arrangement used in making the fluorescence comparisons

suitable iodine acceptor. It was desired if possible to find a reaction which involved only the direct addition of an activated iodine molecule onto a double bond. We finally found that iodine activated by green light combines with hexene at a reasonable rate, while the thermal reaction is not troublesome since at low pressures the equilibrium is in the direction of almost complete dissociation. The hexene used was prepared by the dehydration of hexyl alcohol with 80 per cent phosphoric acid. It had a

boiling point of 68–70°C. and probably consisted principally of the straight-chain form having the double bond between the second and third carbon atoms.

The experiments on the iodine separation were conducted as follows. A tubular vessel of pyrex glass, having at one end a plane window and at the other end a conical "light-trap," was evacuated and then filled with iodine at about 0.17 mm. pressure, and then with hexene at about 6 mm. partial pressure. The tube was then subjected to the intense light from two Cooper-Hewitt glass mercury arcs, using a filter of 0.05 molal potassium dichromate 2 cm. in thickness to cut off all radiations on the violet side of the green mercury line. The lamps were run at considerably below the rated capacity, and were cooled by a blast of air to keep the emission lines as narrow as possible.

After 24 hours about half the iodine was found to have reacted. Tests were then made as follows to determine whether any separation had been effected. A comparison tube was filled with hexene at the same pressure as before, but with the iodine partial pressure adjusted to give the same purple color as that of the reaction tube.

The fluorescence in the reaction and comparison tubes was then compared as represented in the accompanying figure. The light from an intense source was concentrated with strong lenses and passed through the sides of the tubes. On looking down through the plane windows, cones of orange fluorscence were visible. The two tubes were placed side by side, and when equally illuminated comparisons were made of the intensity of fluorescence in the two. These comparisons were necessarily rather qualitative as the fluorescence was weak due to the quenching effect of the hexene.

When the white light from a tungsten lamp was used for excitation, the fluorescence had the same intensity in the two tubes within the experimental error. But when a mercury arc burning at low pressure was used as source of excitation the fluorescence in the comparison tube was considerably stronger. The difference was not great, but quite definite.

The purple color and white-light fluorescence indicate that we have about the same amount of iodine in the two tubes; but evidently there is a difference in certain properties. We believe that we are justified in concluding that in the reaction tube the equilibrium existing in ordinary iodine gas has been materially disturbed, and that an iodine has been produced which contains a higher percentage of the type of molecule which cannot absorb the mercury green line than in the normal mixture.

As to why a more complete separation was not obtained there may be several answers. In the first place it is possible that the rate at which equilibrium between the two forms of iodine is restored is not negligibly

small in comparison with the rate of the photochemical reaction. Secondly, the photochemical reaction may not be as simple as one might hope and may involve the production of iodine atoms at some stage. We hope later to investigate the rate and nature of the reaction, as the possibility of distinguishing between two kinds of molecule gives a new tool for this purpose, especially useful in the case of chain reactions.

However, a final experiment, using a more concentrated dichromate filter, indicated that we may be able to improve our results considerably. In this event it is expected to compare the properties of the two types of iodine and to measure the rate at which equilibrium between the two is restored. Some qualitative experiments along this line, which need to be confirmed, indicate that equilibrium is restored rather slowly and several days at least are required. Condensation and re-evaporation of the iodine does not seem to hasten the attainment of equilibrium appreciably, indicating the persistence of the two molecular forms in the solid state.

[1] Wood and Loomis, *J. Frank. Inst.*, **205**, 481 (1928).

NOTE ON THE EQUATION OF STATE EXPLICIT IN THE VOLUME

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Communicated October 29, 1930

Kinetic considerations always lead to an equation of state giving the pressure as a function of the concentration and temperature, but for many purposes it is more convenient to know the molal volume as a function of the pressure and temperature. Beattie[1] has indicated how the constants of an equation, similar in form to the Beattie-Bridgeman equation,[2] may be obtained from p,V,T data. Often, however, the labor of this determination is not compensated by the added convenience.

Beattie[1] has also shown how an approximation sufficient for many purposes may be obtained making use of the constants already obtained for the Beattie-Bridgeman equation. The treatment is the same for any equation which may be expressed in the virial form. So expressed the Beattie-Bridgeman equation has four terms:

$$p = RT(n/V) + \beta(n/V)^2 + \gamma(n/V)^3 + \delta(n/V)^4 \qquad (1)$$

p is the pressure, R the gas constant, T the absolute temperature, n the

number of moles and V the volume. The coefficients β, γ, δ are functions of the temperature.[2] Multiplying by V/np gives:

$$V/n = RT/p + \frac{\beta}{p}(n/V) + \frac{\gamma}{p}(n/V)^2 + \frac{\delta}{p}(n/V)^3. \qquad (2)$$

Beattie replaces n/V on the right-hand side by the perfect gas relation

$$n/V = p/RT \qquad (3)$$

giving

$$V/n = RT/p + \beta/RT + \gamma p/(RT)^2 + \delta p^2/(RT)^3. \qquad (4)$$

In many cases a much better approximation of (1) may be obtained by the repeated use of (2) itself rather than (3) to give the value of n/V, and expressing the result as a power series in $p/$ stopping at the term in p^2, which gives:

$$V/n = RT/p + \beta/RT + [\gamma/(RT)^2 - \beta^2/(RT)^3]p + [\delta/(RT)^3 - 3\beta\gamma/(RT)^4 + 2\beta^3/(RT)^5]p^2. \qquad (5)$$

Equation (5) is as simple as (4) except for the computation of the extra terms in the coefficients.

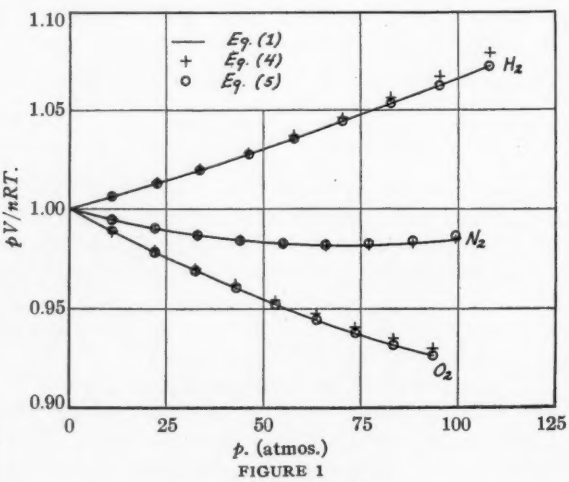

FIGURE 1

Pressure volume products.

The accuracy of the two equations in reproducing (1) is compared in figure 1 for some nearly perfect gases, and in figure 2 for some less perfect ones. The full lines are pV/nRT at 0° C. from (1), the crosses are the

corresponding values from (4) and the circles from (5). The radii of the circles are 0.002 in figure 1 and 0.004 in figure 2. Equation (5) has a considerable advantage in every case except nitrogen, and there the deviation is less than 0.3% at 100 atmospheres.

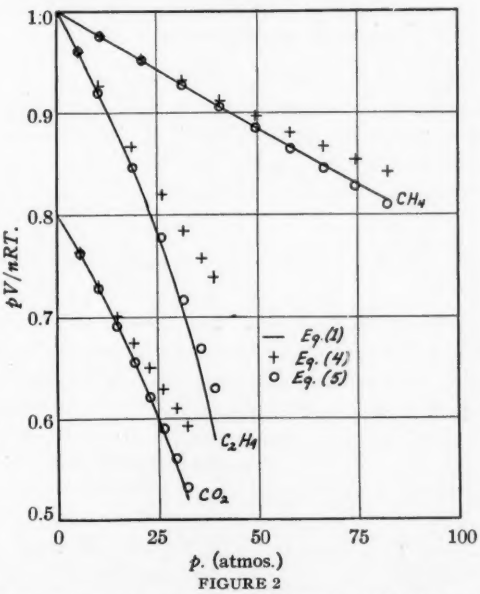

FIGURE 2

Pressure volume products. Ordinates for CO_2 displaced 0.2 units.

Even for the more perfect gases, equation (5) is not satisfactory for extremely high pressures, and it should not be used over a higher pressure range without testing its accuracy. The test can best be made by computing p from (1) for some arbitrary value of V/n, and using this value of p to compute V/n from (5). Generally a single test at the highest concentration is sufficient.

[1] Beattie, *Proc. Nat. Acad. Sci.*, **16**, No. 1, pp. 14–19 (1930).

[2] Beattie and Bridgeman, *J. Am. Chem. Soc.*, **49**, 1665 (1927).

ON THE INTENSITY OF TOTAL SCATTERING OF X-RAYS BY MONATOMIC GASES

BY Y. H. WOO

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Communicated November 14 1930

Raman (*Indian J. Phys.* **3,** 357, 1928) and A. H. Compton (*Phys. Rev.*, **35,** 925, 1930) have calculated according to classical electrodynamics the scattering of x-rays by an atom in which the electrons are arranged with random orientation and with arbitrary radial distribution. The intensity of the x-rays scattered at an angle θ to a distance R is given by

$$I_\theta = \frac{Ie^4(1+\cos^2\theta)}{2m^2R^2C^4}\left\{ Z + (Z^2 - Z)\left[\int_0^a 4\pi r^2 p(r)\,\frac{\sin kr}{kr}\,dr\right]^2\right\}, \quad (1)$$

where I is the intensity of the primary beam, $k = (2\pi/\lambda)\sin\theta/2$, $4\pi r^2 p(r)dr$ is the probability that any electron shall lie between r and $r + dr$ from the nucleus, a is the maximum radius of the atom and Z, e, m and c have their usual significance. In comparison with Wentzel's quantum theory of x-ray scattering, Compton has separated I_θ into two parts, namely, I_1 representing the intensity of coherent scattering and I_2 the intensity of incoherent scattering, where

$$I_1 = \frac{Ie^4(1+\cos^2\theta)}{2m^2R^2C^4}\,Z^2F^2, \quad I_2 = \frac{Ie^4(1+\cos^2\theta)}{2m^2R^2C^4}\,Z(1-F^2)$$

and

$$F = \int_0^a 4\pi r^2 p(r)\,\frac{\sin kr}{kr}\,dr.$$

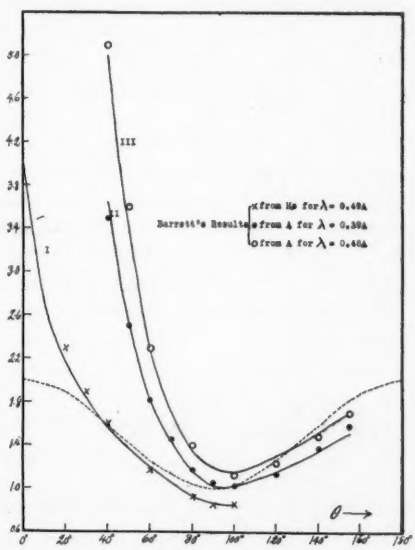
Raman has come to the same conclusion by simple classical consideration. When corrected for the change of wave-length, Eq. (1) becomes (cf. Compton, loc. cit.)

$$I_\theta = I_1 + I_2[1 + \gamma(1-\cos\theta)]^{-3} \quad (2)$$

where $\gamma = h/mc\lambda$.

If instead of the probable position of a single electron, we regard $Z4\pi r^2 p(r)dr$ as the probable number of electrons between r and $r + dr$, we see therefore that the calculation of the intensity of the total scattering entirely depends on the evaluation of the radial charge distribution of the electrons in the atom. It is well known that Thomas (*Proc. Camb. Phil. Soc.*, **23,** 542, 1927) and Fermi (*Zeit. Phys.*, **48,** 73, 1928) have independently derived an approximate expression for the charge distribution of the electrons in the atom by considering the electrons as a degenerate gas

surrounding the nucleus (degenerate in the sense of Fermi and Dirac), an idea which seems to be in accordance with the atomic model postulated by Raman and Compton in deducing equation (1). If the charge density of the electrons in the atom evaluated by Thomas-Fermi's method is substituted in place of $Zp(r)$ in equation (2), the intensity of total scattering for any scattering angle can be numerically calculated. Owing to the interference effect due to neighboring atoms in diatomic molecules, we expect equation (2) to be directly applicable only to the scattering of x-rays by monatomic gases and vapors. A calculation is made of the intensity of the scattering of x-rays by helium and argon and the results are compared with the experimental data recently obtained by Barrett (*Phys. Rev.*, **32,** 22, 1928) in the figure, where the scattering per electron is plotted against the scattering angle θ. While the curve I represents the scattering from helium for a wave-length equal to 0.49Å, the curves II and III represent the scattering from argon for wave-lengths equal to 0.40Å and 0.48Å, respectively. The classical theory of J. J. Thomson for the scattering from a single electron is plotted as the broken curve marked *C*. Since Barrett's measurements give relative values of scattering per electron for different angles and for different gases, but not absolute values, so in each case the experimental data have been multiplied by an arbitrary factor throughout. It is seen that the agreement between theory and experiment seems to be very satisfactory.



Recently Waller and Hartree (*Proc. Roy. Soc.*, **A124,** 119, 1929) have investigated theoretically the intensity of total scattering of x-rays by atoms of a monatomic gas on the basis of quantum mechanics. They have developed a formula for the value of R the ratio of the total intensity of scattering by a many-electron atom to the classical intensity of scattering by a free electron and concluded that the quantity R is a function of $\sin\frac{\theta}{2}/\lambda$ only. As pointed out by Waller and Hartree, a strict comparison

of their results with Barrett's observations is not possible, as for the short wave-lengths used by Barrett "relativity effects" are appreciable, whereas they are neglected in the derivation of the theoretical formula. Owing to the factor introduced to correct for the change of wave-length, equation (2) shows that the quantity R as defined by Waller and Hartree is not a function of $\sin\frac{\theta}{2}/\lambda$ only, a result not in agreement with the conclusion drawn by these authors. A comparison of Waller and Hartree's results for argon with those of the present calculation also indicates that the absolute values of the intensity of scattering per electron for large angles of scattering given by these authors are much higher than those calculated according to Equation (2). Unfortunately no experimental results are available to decide this point.

Finally it may be remarked that by the method outlined above the scattering by all monatomic gases and vapours can be approximately estimated. The writer has numerically evaluated the scattering from He, A, Ne, Kr, Na, K and Hg for Mok_α and CuK_α radiation and experiments are in progress to test these results.

A detailed account of this work will be published elsewhere.

IRROTATIONAL MOTION OF A COMPRESSIBLE INVISCID FLUID

By H. Bateman

California Institute of Technology

Read before the Academy, September 23, 1930

1. *Variational Principles.*—Let us assume that in the free two-dimensional irrotational motion of a compressible inviscid fluid the "density of mechanical energy" $p + {}^1/_2\,\rho\,q^2$ is an assigned differentiable function $f(\rho)$ of the density ρ of the fluid. The pressure p is then given by the equation

$$p = f(\rho) - ({}^1/_2)\,\rho q^2 = f(\rho) - ({}^1/_2)\rho\left[\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2\right] \qquad (1)$$

where ϕ is the velocity potential. In steady motion the pressure energy

$$P = \int\int p\,dxdy \qquad (2)$$

will be an extremum (sometimes a maximum) for certain allowable variations of ρ and ϕ when, among other conditions, the relations

$$O = f'(\rho) - {}^1/_2\,q^2 \qquad (3)$$

$$\rho \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}, \quad \rho \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} \tag{4}$$

are satisfied, ψ being a function analogous to that introduced by Haar[1] in his treatment of two-dimensional variational problems. In the hydrodynamical problem ψ is the stream-function and when the function ϕ is assumed to have continuous second derivatives the elimination of ψ gives the equation of continuity

$$\frac{\partial}{\partial x}\left(\rho \frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho \frac{\partial \phi}{\partial y}\right) = 0. \tag{5}$$

This equation and (3) together represent the differential equations of Euler and Lagrange for this particular variation problem. The variational problem itself is a simple modification of one used by Clebsch.[2]

To find when the variation problem is regular we write $\rho + \epsilon\tau$ in place of ρ and $\phi + \epsilon x$ in place of ϕ, ϵ being a small quantity. The coefficient of $\frac{1}{2}\epsilon^2$ in the new expression for P is then

$$\int\int dxdy \left\{\tau^2\left[f''(\rho) + \frac{1}{\rho}\left(\frac{\partial \phi}{\partial x}\right)^2 + \frac{1}{\rho}\left(\frac{\partial \phi}{\partial y}\right)^2\right] - \rho\left[\frac{\partial x}{\partial x} + \frac{\tau}{\rho}\frac{\partial \phi}{\partial x}\right]^2 - \rho\left[\frac{\partial x}{\partial y} + \frac{\tau}{\rho}\frac{\partial \phi}{\partial y}\right]^2\right\}.$$

Now when use is made of (3) the pressure is given by the equation

$$p = f(\rho) - \rho f'(\rho), \tag{6}$$

and if c denotes the "local velocity of sound" we have

$$c^2 = \frac{dp}{d\rho} = -\rho f''(\rho). \tag{7}$$

The coefficient of $\frac{1}{2}\epsilon^2$ in P is thus negative when $q^2 < c^2$ at all points of the region of integration. If, on the other hand, there are places where, in the extremal motion, $q^2 < c^2$ we cannot assert that the pressure energy is a maximum.

It should be noticed that equation (7) gives

$$\int \frac{dp}{\rho} = -f'(\rho), \tag{8}$$

and so we have the energy-equation

$$\int \frac{dp}{\rho} + \tfrac{1}{2} q^2 = 0. \tag{9}$$

This is in accordance with the well-known fact that the mechanical energy does not represent the whole energy, for thermal energy must be taken into

consideration. Equations (4) may be derived from a variational principle of the simple two-dimensional type considered by Haar by expressing p in terms of q with the aid of equation (3). The variational problem is then to make $\delta I = 0$ where

$$I = \int\int G(q^2)dxdy \tag{10}$$

and

$$q^2 = \left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2, \tag{11}$$

the function ϕ being given certain allowable variations. This variational problem has already been considered by the author[3] and has been shown to be regular when the extremal function is such that $q^2 < c^2$ at all points of the region of integration. It should be mentioned that

$$G'(q^2) = -\,{}^1/_2\rho. \quad G''(q^2) = -\frac{1}{4f''(\rho)}, \quad \frac{G'(q^2)}{G''(q^2)} = 2\rho f''(\rho) = -2c^2. \tag{12}$$

When the function $G(q^2)$ is given, the function $f(\rho)$ may be obtained by solving the differential equation

$$f(\rho) - \rho f'(\rho) = G[2f'(\rho)]. \tag{13}$$

This is an equation of Clairaut's type and its general solution is

$$f(\rho) = C\rho + G[2C] \tag{14}$$

where C is an arbitrary constant. The general solution is not, however, of much interest from the physical standpoint; it is the singular solution which is needed and this is obtained by eliminating C from the equations

$$\begin{aligned} f(\rho) &= C\rho + G[2C] \\ O &= \rho + 2G'[2C]. \end{aligned} \tag{15}$$

In particular, if

$$G(q^2) = A + \sqrt{(1+q^2)},$$

where A is a constant, we find that

$$\begin{aligned} f(\rho) &= A - {}^1/_2(\rho + 1/\rho) \\ p &= f(\rho) - \rho f'(\rho) = A - 1/\rho. \end{aligned}$$

When equations (3) and (4) are satisfied the first variations δP and δI may be expressed in the form

$$\int\int \frac{\partial(\psi,\delta\phi)}{\partial(x,y)}\,dxdy$$

and may be replaced by a line integral

$$\int \delta\phi.d\psi$$

round the boundary of the region of integration. The line integral vanishes when ψ is constant on one part of the boundary and $\delta\phi = 0$ on the other part.

Equations (3) and (4) may also be obtained from a variational principle of type $\delta J = 0$, where

$$J = \int\int dxdy \left[f(\rho) + \frac{1}{2\rho}\left\{ \left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial y}\right)^2 \right\} \right], \tag{16}$$

and the quantities ρ and ψ are to be given certain allowable variations. When (3) and (4) are satisfied the expression for J can be written in the form

$$J = \int\int dxdy \left[p + \frac{\partial(\phi,\psi)}{\partial(x,y)} \right], \tag{17}$$

and may be transformed into

$$P - \int \phi d\psi, \tag{18}$$

where the line integral is taken round the boundary of the region of integration. Thus J does not generally represent the pressure energy but it may do when the line integral vanishes on account of the assigned boundary conditions.

The integral which is to be varied may be expressed in another form by writing Q for the density of momentum, i.e.,

$$Q^2 = \left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial y}\right)^2 = \rho^2 q^2 = 2\rho^2 f'(\rho), \tag{19}$$

and using this equation to express the integrand in terms of Q alone. It should be noted that this equation may fail to give a real expression for ρ as a function of Q. A critical case occurs when the curves

$$Y = Q^2/2\rho^2, \quad Y = f'(\rho) \tag{20}$$

touch, that is, when

$$f''(\rho) = -Q^2/\rho^3 \tag{21}$$

or $c^2 = q^2$. When the equation does give real values of ρ these will generally be of two types, one for which $q^2 < c^2$ and another for which $q^2 > c^2$. When $q^2 = c^2$ the relations

$$q^2 = 2f'(\rho), \quad c^2 = -\rho f''(\rho) \tag{22}$$

indicate that c^2 has a special value a^2 and ρ a corresponding special value σ.

The partial differential equation for ψ may be written in the form

$$(Q^2 - c^2\rho^2)\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) = Q\left(\frac{\partial Q}{\partial x}\frac{\partial\psi}{\partial x} + \frac{\partial Q}{\partial y}\frac{\partial\psi}{\partial y}\right). \tag{23}$$

from which it appears that in the critical case when $Q^2 = c^2\rho^2 = a^2\sigma^2$ the

curve $Q^2 = a^2\sigma^2$ is orthogonal to the streamlines and is therefore an equipotential curve. An exceptional case arises, however, when Q^2 is a function of ψ only. The right-hand side of (23) then vanishes because Q^2 is a maximum for the special value of ψ which gives $Q^2 = a^2\sigma^2$.

When the two variational problems $\delta I = 0$, $\delta J = 0$ are both real and one is regular the other is also regular. In the particular case when $G(q^2) = (1 + q^2)^{1/2}$ the variational problem $\delta I = 0$ is equivalent to that of finding a minimal surface which satisfies certain boundary conditions. The associated variational problem $\delta J = 0$ is then specified by the function

$$H(Q)^2 = (1 - Q^2)^{1/2} \tag{24}$$

and the condition for regularity is easily seen to be $Q^2 < 1$, which is satisfied when the variational problem is real. It should be noticed that in this case

$$Q^2 = \frac{q^2}{1 + q^2} \tag{25}$$

so although $\frac{\partial\phi}{\partial x}$ and $\frac{\partial\phi}{\partial y}$ may be allowed to have any real values in the original variation problem $\delta I = 0$, the derivatives in the variational problem $\delta J = 0$ are restricted by the condition $Q^2 < 1$.

When the variational problems $\delta I = 0$, $\delta J = 0$ are real and regular and the extremal functions ϕ and ψ have continuous second derivatives the partial differential equations for ϕ and ψ are of elliptic type. It then follows from a remark[4] made by Radó that the extremal functions satisfy the inequalities

$$\begin{aligned} \frac{\partial^2\phi}{\partial x^2}\,\frac{\partial^2\phi}{\partial y^2} &\leqq \left(\frac{\partial^2\phi}{\partial x\partial y}\right)^2 \\ \frac{\partial^2\psi}{\partial x^2}\,\frac{\partial^2\psi}{\partial y^2} &\leqq \left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2. \end{aligned} \tag{26}$$

Indeed, if $Ar + 2Bs + Ct = 0$, where $AC > B^2$, we may write

$$4B^2(s^2 - rt) = (Ar + Ct)^2 - 4B^2rt$$

and it is easily seen that the quadratic form on the right is always positive. The surfaces represented by the equations $z = \phi(x,y)$ $z = \psi(x,y)$ are thus saddle surfaces and we have the hydrodynamical theorem that when $q^2 < c^2$ throughout the region of integration in the variation problems we have the inequality

$$\frac{\partial(u,v)}{\partial(x,y)} \leqq 0 \tag{27}$$

at all points of this region.

Some other interesting properties of regular two-dimensional variation problems are given in Radó's paper.

Spiral Flow.—Q^2 is a function of ψ only in the case when ψ depends only on r, the distance from the origin. Using polar coördinates (r,θ) we proceed to study Taylor's solution[5]

$$\phi = A\theta + BF(r), \tag{28}$$

in which A and B are arbitrary constants and $F(r)$ is a function to be determined, taking the general case, when the function $f(\rho)$ is not restricted, to be of the adiabatic type. Noting that $\frac{\partial\phi}{\partial\theta} = A$ we see that q^2 depends only on r, hence ρ also depends only on r and the differential equation for ϕ takes the form

$$\rho\left[r\frac{\partial^2\phi}{\partial r^2} + \frac{\partial\phi}{\partial r}\right] + r\frac{\partial\rho}{\partial r}\frac{\partial\phi}{\partial r} = 0. \tag{29}$$

This may be integrated giving

$$\rho r\frac{\partial\phi}{\partial r} = \tau A \tag{30}$$

where τ is a constant. In Taylor's notation

$$\nu a^2 = \frac{1}{r^2}\left(\frac{\partial\phi}{\partial\theta}\right)^2, \quad \eta a^2 = \left(\frac{\partial\phi}{\partial r}\right)^2. \tag{31}$$

The relation takes the form

$$\rho^2\eta = \tau^2\nu \tag{32}$$

or

$$\nu a^2 = \frac{\rho^2 q^2}{\rho^2 + \tau^2}. \tag{33}$$

The solution of Taylor's differential equation

$$\frac{d\eta}{d\nu} = \frac{\eta}{\nu}\,\frac{\gamma + 1 + (3-\gamma)\nu - (\gamma-1)\eta}{\gamma + 1 - (\gamma-1)\nu - (\gamma+1)\eta}, \tag{34}$$

which corresponds to the case in which

$$f'(\rho) = \frac{a^2}{2(\gamma-1)}\left[\gamma + 1 - 2\,(\rho/\sigma)^{\gamma-1}\right], \tag{35}$$

where γ is a constant, is thus obtained by eliminating ρ from the equations

$$\nu a^2 = \frac{2\rho^2 f'(\rho)}{\rho^2 + \nu^2}$$
$$\eta a^2 = \frac{2\tau^2 f'(\rho)}{\rho^2 + \tau^2}. \tag{36}$$

It will be noticed that ν is a maximum when

$$\frac{2}{\rho} + \frac{f''(\rho)}{f'(\rho)} = \frac{2\rho}{\rho^2 + \tau^2}; \tag{37}$$

that is, when

$$\nu a^2 = 2f'(\rho) + \rho f''(\rho) = q^2 - c^2 \tag{38}$$

or $\eta a^2 = c^2$. The radial velocity is then equal to the local velocity of sound, thus Bickley's result (mentioned by Taylor) holds also in the general case. It appears that ρ is imaginary when r has a value less than that corresponding to the maximum value of ν and that when r has a value for which ρ can be real there are two possible values of ρ.

It should be noticed that when ν is a maximum $d\nu/d\rho$ is zero and $d\rho/d\nu$ infinite. Considered as a function of ν the density ρ has a branch point when ν is a maximum. For the corresponding value of r, then, two streamlines corresponding to different functional expressions $\rho = R(r)$ come together and touch.

3. *Motion with Velocity Greater Than the Local Velocity of Sound.*—An interesting type of motion is obtained by writing

$$u = A(s), v = B(s), \rho = F(s). \tag{39}$$

The equation $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$ is then satisfied if

$$B'(s) \frac{\partial s}{\partial x} = A'(s) \frac{\partial s}{\partial y}, \tag{40}$$

and this equation may be satisfied by writing

$$xA'(s) + yB'(s) = C'(s), \tag{41}$$

where $C'(s)$ is an arbitrary continuous function. An appropriate expression for ϕ is now

$$\phi = xA(s) + yB(s) - C(s), \tag{42}$$

and the equation of continuity

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{43}$$

will be satisfied if the function $F(s)$ is chosen so that

$$F'(s)\,[A(s)A'(s) + (Bs)B'(s)] + F(s)\,[\{A'(s)\}^2 + \{B'(s)\}^2] = 0. \quad (44)$$

When the functions $A(s)$, $B(s)$, $C(s)$ are assigned the function $f(\rho)$ cannot be chosen arbitrarily. Indeed, if $f(\rho) = G(s)$ we have

$$f'(\rho)\ F'(s) = G'(s) \quad (45)$$

and since

$$2f'(\rho) = u^2 + v^2 = [A(s)]^2 + [B(s)]^2, \quad (46)$$

we have the equation

$$2G'(s) = F'(s)\ [\{A(s)\}^2 + \{B(s)\}^2] \quad (47)$$

for the determination of $G(s)$. When the function $G(s)$ has been found the pressure is given by the equation

$$p = G(s) - F(s)\,\frac{G'(s)}{F'(s)}, \quad (48)$$

while

$$C^2 = \frac{dp}{d\rho} = \frac{F(s)\ [F''(s)G'(s) - F'(s)G''(s)]}{[F'(s)]^3}$$

$$= \frac{[A(s)\,A'(s) + B(s)\,B'(s)]^2}{\{A'(s)\}^2 + \{B'(s)\}^2} \quad (49)$$

$$q^2 - c^2 = \frac{[A(s)\,B'(s) - A'(s)\,B(s)]^2}{\{A'(s)\}^2 + \{B'(s)\}^2} > 0. \quad (50)$$

The inequality shows that the present type of motion can exist only in a region where the fluid velocity is everywhere greater than the local velocity of sound. The motion has the interesting peculiarity that u, v, and ρ have constant values on each one of a certain family of straight lines given by the equation (41). An equipotential curve may be regarded as the envelope of a family of lines given by an equation of type (42). For each such family of lines there is a relation between p and characteristic of a fluid for which steady motion is possible with such a value of ϕ.

It should be noticed that in this type of motion we have

$$\frac{\partial(u, v)}{\partial(x, y)} = 0$$

and also

$$q/c = \text{Sin}\ \xi$$

where Mach's angle ξ is the complement of the angle between the lines (41) and (42).

4. *The Circulation round an Airfoil.*—The analysis given by Glauert[6] is readily extended to the case in which the function $f(\rho)$ is of general form. Assuming that at a great distance r from the origin the component velocities u,v are approximately

$$u = V(\cos\theta + A/r), \quad v = -V(\sin\theta + B/r) \tag{51}$$

where A,B are functions of the polar angle θ and U is a constant, we have to this order

$$q^2/V^2 = 1 + 2(A\cos\theta + B\sin\theta)/r = 1 + 2\theta/r, \text{ say.} \tag{52}$$

We now assume that ρ differs only slightly from its value ρ_0 at infinity while p differs only slightly from the associated value p_0 and we write

$$C^2 = \frac{dp}{d\rho} = -\rho f''(\rho), \quad a^2 = -\rho_0 f''(\rho_0)$$

$$\tfrac{1}{2}q^2 = f'(\rho), \quad \tfrac{1}{2}V^2 = f'(\rho_0)$$

$$\frac{q^2}{U^2} = \frac{f'(\rho)}{f'(\rho_0)} = 1 + (\rho - \rho_0)f''(\rho_0)/f'(\rho_0)$$

$$= 1 + 2a^2(\rho_0 - \rho)/\rho_0 V^2 = 1 + 2(\rho_0 - \rho)/\lambda\rho_0,$$

where $\lambda = V^2/a^2$.

Therefore

$$\rho/\rho_0 = 1 - \lambda\Theta/r. \tag{53}$$

Also

$$p - p_0 = -\rho_0 f''(\rho_0)(\rho - \rho_0) = a^2(\rho - \rho_0)$$

$$= V^2(\rho - \rho_0)/\lambda = -\rho_0 V^2\Theta/r.$$

Therefore

$$\frac{r}{\rho}\frac{\partial\rho}{\partial r} = \frac{\lambda\Theta}{r}, \quad \frac{1}{\rho}\frac{\partial\rho}{\partial\theta} = -\rho_0 V^2\Theta'/(r). \tag{54}$$

The analysis then proceeds as in Glauert's paper and it is found that

$$A = 0, \quad 2\pi VB(1 - \lambda\sin^2\theta) = K\sqrt{(1 - \lambda)} \tag{55}$$

where U is the velocity of flow past the aerofoil and K is the circulation round it. Thus we have approximately

$$u = U \cos\theta$$

$$v = -U \sin\theta - \frac{K}{2\pi r} \frac{\sqrt{1-\lambda}}{1-\lambda \sin^2\theta} \tag{56}$$

and it may be proved as in Glauert's paper that when the above approximations are valid the effect of compressibility is to increase the lift in the ratio $1:\sqrt{1-\lambda}$, a result also given by Prandtl[7] and Ackeret.[8]

5. *The Effect of Varying the Function $f(\rho)$.*—Let us replace $f(\rho)$ by $f(\rho) + \epsilon F(\rho)$ where ϵ is so small that terms involving ϵ^2 may be neglected. The effect of this variation is to change ρ into $\rho + \epsilon\sigma$, ϕ into $\phi + \epsilon\chi$ and ψ into $\psi + \epsilon\omega$ in equations (2) and (3). The variational equations are thus

$$\sigma f''(\rho) + F'(\rho) = \frac{\partial\phi}{\partial x}\frac{\partial\chi}{\partial x} + \frac{\partial\phi}{\partial y}\frac{\partial\chi}{\partial y}$$

$$\frac{\partial\omega}{\partial y} = \sigma\frac{\partial\phi}{\partial x} + \rho\frac{\partial\chi}{\partial x}, \quad -\frac{\partial\omega}{\partial x} = \sigma\frac{\partial\phi}{\partial y} + \rho\frac{\partial\chi}{\partial y}. \tag{57}$$

In the case when the second derivatives of ϕ, χ and ω exist the differential equation for χ is

$$\frac{\partial}{\partial x}\left[\sigma\frac{\partial\phi}{\partial x} + \rho\frac{\partial\chi}{\partial x}\right] + \frac{\partial}{\partial y}\left[\sigma\frac{\partial\phi}{\partial y} + \rho\frac{\partial\chi}{\partial y}\right] = 0 \tag{58}$$

where σ is given by (57). The equation is of elliptic type when

$$\left[\rho f''(\rho) + \left(\frac{\partial\phi}{\partial x}\right)^2\right]\left[\rho f''(\rho) + \left(\frac{\partial\phi}{\partial y}\right)^2\right] > \left(\frac{\partial\phi}{\partial x}\frac{\partial\phi}{\partial y}\right)^2, \tag{59}$$

that is, when $c^2(c^2 - q^2) > 0$. The equation is thus of elliptic type when the differential equation for ϕ is of elliptic type.

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A STUDY OF INDEFINITELY DIFFERENTIABLE AND QUASI-ANALYTIC FUNCTIONS

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1. ON SERIES OF THE FORM $\Sigma x_i f(a_i x)$. We introduce the

Definition.—An entire function $f(x)$, such that the inequalities $|f(x)|$, $|f^{(p)}(x)| < h$ $(p = 1, 2, \ldots)$ hold for all real values of x, will be said to be of class A.

THEOREM I. The series

$$\zeta(x) = \sum_{k=1}^{\infty} x_k f(a_k x), \quad |a_{k+1}| > |a_k|, \quad |a_1| \geqq 1, \quad |a_k| \longrightarrow \infty \qquad (1)$$

(a_k real), where $f(x)$ is of class A, is indefinitely differentiable for all real x and represents a function with assigned initial values $F_{(0)}^{(n)}$ $(n = 0, 1, \ldots)$ at $x = 0$ provided the following conditions hold. The series $\Sigma n^2 |F_{(0)}^{(n)}/f_{(0)}^{(n)}|$ converges; the x_k are given by

$$x_k = \frac{1}{2\pi i} \int_C \frac{\varphi(z)\,\theta(z)\,\psi(z)\,dz}{(z-a_k)\,\varphi^{(1)}(a_k)\,\theta(a_k)}. \qquad (2)$$

Here $\psi(z) = \Sigma F_i/z^{i+2}$ $(F_{(0)}^{(i)}/f_{(0)}^{(i)} = F_i)$, C is a suitable circle with the origin for the center; $\varphi(z)$ has the a_k for simple and the only zeros; $\theta(x)$ is a suitable entire function.

There is a similar theorem for the case when $f(x)$ is even. The results of this kind are based on certain work of Borel.[1]

Another type of theorems is obtained for the case $a_i = i$, in connection with certain Fourier expansions. For instance:

THEOREM II. Let $p(x)$ be an even function of class A; let the $F_{(0)}^{(2n)}$ be an assigned set of constants such that $\psi(x)$ $(= \Sigma(-1)^i F_{(0)}^{(2i)} x^{2i}/p_{(0)}^{(2i)}(2i)!)$ and all the derived series of $\psi(x)$ converge for $0 \leqq x \leqq \pi$. Then the series $b_0 p(0)/2 + \sum_1^{\infty} b_k p(kx)$ is indefinitely differentiable for all real x and represents a function $\zeta(x)$ for which $\zeta_{(0)}^{(2n)} = F_{(0)}^{(2n)}$, provided the b_k are the Fourier constants of the cosine expansion of a function $g(x) = \psi(x) + e^{-1/x^2}\theta(x)$. Here $\theta(x)$ is a suitably determined power series.

Utilizing a result of Poincaré[2] we have the

THEOREM III. Let $p(x)$ be even and $q(x)$ odd, both of class A. Let $r(x)$, $r_1(x)$ be suitable entire functions. Then the series

$$\zeta(x) = \frac{x_0\ p(0)}{2} + \sum_{n=1}^{\infty}[x_n^1 q(nx) + x_n p(nx)], \tag{3}$$

$$x_n^1 = \frac{r_1(n^2)}{\pi}\int_{-\pi}^{\pi} L(x)\sin nxdx;\ x_n = \frac{r(n^2)}{\pi}\int_{-\pi}^{\pi} L(x)\cos nxdx,$$

($L(x)$ a properly determined function) is indefinitely differentiable for all real x and represents a function with zero initial values at $x = 0$.

2. ON BOREL MONOGENIC FUNCTIONS. Concerning notation and the definition of Borel monogenic functions reference is made to Borel's book on monogenic functions.[3] We shall note only that they form an extension of the analytic class; they constitute a special class of quasi-analytic functions[4] of a complex variable.

If $f(z)$ is Borel monogenic in $C^{(p)}$ then interior $\Gamma^{(p)}$ we have, as proved by Borel,

$$f(x) = \frac{1}{2\pi i}\int_K \frac{f(z)dz}{z-x} - \sum_k{}^1 \frac{1}{2\pi i}\int_{\zeta_k^{(p)}} \frac{f(z)dz}{z-x} \tag{A}$$

$$= B(f;\ K, C^{(p)}, z;\ x).$$

Moreover,

$$\int_K f(z)dz - \sum_k{}^1 \int_{\zeta_k^{(p)}} f(z)dz = 0. \tag{B}$$

Here the $\zeta_k^{(p)}$ are the circles used in defining $C^{(p)}$ (a perfect discontinuous set) and K is a closed simple curve interior to $C^{(p)}$. On the basis of these formulas I derive certain consequences. Some of the theorems are the following.

THEOREM I. If $f(z)$ is Borel monogenic in $C^{(p)}$, then interior to $\Gamma^{(p)}$

$$f(x) = \sum_{n,m=1}^{\infty} \frac{b_{n,m}^{(p)}}{(x-\alpha_n)^m} + \text{analytic function} \tag{1}$$

(the α_n may be dense in a portion of the plane).

THEOREM II. A series of the form (1) is Borel monogenic in $C^{(p)}$ if the series

$$\sum_{n,m} \frac{|\ b_{n,m}\ |\ g_1(m)}{(r_n^{(p)})^{2m+1}}$$

converges for some $h(r_n^{(p)}$ is the radius of $\zeta_n^{(p)}$; $g_1(m) = \sum_0^{m-1} e_i h^i$; $e_i = C_{i+1}^m + C_{i+1}^{m-1} + \ldots + C_{i+1}^{i+1})$.

THEOREM III. Let $f(z)$ form a family of Borel monogenic functions defined and bounded as a set in $C^{(p)}$. In $\Gamma_1^{(p-1)}$ the functions of the family satisfy a condition of equi-continuity.

By means of (B) theorems are derived on zeros and poles of Borel monogenic and meromorphic functions.

Further, a process is given for quasi-analytic continuation for the case when two Borel monogenic functions have the same values on a given rectifiable arc.

Finally, a number of results is obtained for functions $h(x,y)$ (x,y real) which exist in regions over which a Borel monogenic function may be defined. For such function formulas are established of the type of Green's formulas.

3. SOME PROBLEMS OF REPRESENTATION OF QUASI-ANALYTIC FUNCTIONS. A considerable part of the results of this part of the paper constitutes an extension of some of the theorems of Carleman (see the book already referred to).

Let C_A denote the class of functions $f(x)$ such that $| f^{(i)}(x) | < Rk^iA_i$ ($i = 1, 2, \ldots$; $0 \leqq x \leqq a$). Necessary and sufficient conditions on the A_i under which C_A is a quasi-analytic class are known. In connection with minimizing a certain definite integral, Carleman shows that

$$f(x) = \lim_{n \to \infty} \sum_{i=0}^{n-1} c_i \omega_{n,i}(x) \quad (f_{(0)}^{(i)} = c_i i!), \tag{1}$$

where the $\omega_{n,i}^{(x)}$ depend only on the class. I derive a number of representations by forming the function

$$F(t) = f(xt + t(t-1)p(x,t))$$

and applying to $F(t)$, as function of t, some expansion already known of functions of a suitable class. In the development so derived we let $t = 1 (F(1) = f(x))$. To avoid undue complications the function $p(x,t)$ has to be suitably restricted.

Definition.—An even function $p(x)$ of class A belongs to the subclass A^1, if the relation $\sum a_n p(nx) \equiv 0$, for all real x, implies that $a_n = 0$ ($n = 0,1, \ldots$).

THEOREM I. Let C_α denote the class of functions of the form $F(x) = \sum_0^\infty a_n p(nx)$ ($|a_n| < A\alpha_n$; $p(0) = 1$). If $p(x)$ belongs to A^1 for quasi-analyticity of C_α it is necessary and sufficient that the problem of moments

$$\int_0^\infty x^n \, d\psi(x) = \sum \alpha_i^2 i^{2n} \quad (n = 0,1, \ldots) \tag{2}$$

be determinate. The sufficient part continues to hold when $p(x)$ belongs to A.

Definition.—An even function $p(x)$, of class A, will be said to belong to the subclass B if there exists an odd entire function $g(x)$ so that the function $q(ix) = p(x) + ig(x)$ is such that $\lim_r M(r) = 0$. Here $M(r)$ is the maxi-

mum in θ of $q(re^{i\theta})$ $\left(\frac{\pi}{2} + \epsilon \leqq \theta \leqq \frac{3\pi}{2} - \epsilon\right)$. (An example different from $\cos x$ is $p(x) = \int_a^b \cos(x\varphi(\phi))d\phi$, $0 < \lambda \leqq \varphi(\phi) \leqq 1$, $a \leqq \phi \leqq b$.)

I prove that a function $f(x)$ (of the form $\sum a_n p(nx)$; $|a_n| < A\alpha_n$; $p(x)$ of subclass B; C_α quasi-analytic) is expressible in terms of its initial values $f_{(0)}^{(2\rho)}$ as follows.

$$f(x) = AF(x) - \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} zq(xz)\Omega(z^2)dz \qquad (3)$$

($a > 0$; $F(x) = \Sigma\alpha_n p(nx)$; the continued fraction $\Omega(z)$ is the development of the series $\Sigma(-1)^\rho (AF_{(0)}^{(2\rho)} - f_{(0)}^{(2\rho)}/p_{(0)}^{(2\rho)} z^{\rho+1})$. Here it is assumed that the series $\Sigma 1/\sqrt[2n]{d_n}$ ($d_n = \Sigma\alpha_m m^{2n}$) diverges.

A method involving linear inequalities and applied by de la Vallée Poussin to trigonometric series[5] is extended to series of the form $\sum a_n p(nx)$ ($= f(x)$) ($p(x)$ of class A). The problem solved is the following.

If a set of values $f_{(0)}^{(2n)} = C_{2n}$ belongs to a quasi-analytic class $C_\alpha(|a_n| < A\alpha_n)$ determine the coefficients a_n of the series.

A corresponding problem is solved by a similar method for functions of the form $\sum_{n,m} C_{n,m}/(x-\alpha_n)^m$.

The method of linear inequalities does not enable one to derive necessary and sufficient conditions under which a prescribed set of initial values (C_i) belongs to a given quasi-analytic class. [This problem had been solved only when classes are defined by the law of increase of the moduli of the derivatives (Carleman).]

Under certain mild restrictions, I show that in order that a prescribed set of initial values (C_i) should belong to a given quasi-analytic class $C_\alpha(\Sigma a_n p(nx); |a_n| < A\alpha_n)$ it is necessary and sufficient that the sequence of certain functions (L_i/ϵ_i) should be bounded. These functions are derived in the course of minimizing the quadratic form

$$E_i(y) = \sum_{r=0}^{n_i-1} [C_{2r} - \sum_{n=0}^{p_i-1} p_{(0)}^{(2r)} n^r y_n]^2 + \epsilon_i \sum_{n=0}^{p_i-1} \frac{y_n^2}{\alpha_n^2}$$

In fact, $L_i = \min. E_i(y)$.

A corresponding problem is solved for quasi-analytic classes $C(\alpha; \beta)$:

$$B_n x^n + \sum_{n,m} \frac{A_{n,m}}{(x-\gamma_n)^m} \qquad (|B_n| < A\beta_n;\ |A_{n,m}| < A\alpha_{n,m}),$$

the involved constants being real.

A detailed account of the investigation here outlined will be given in a paper to be published in the *Annals of Mathematics*.

* NATIONAL RESEARCH FELLOW.

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ON THE UNIFIED FIELD THEORY. II

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In our previous note[1] we constructed a system of field equations for the combined gravitational and electromagnetic field which, on the basis of the interpretation of the quantities h_α^i as electromagnetic potentials, led to Maxwell's equations in their exact form at the origin of the local coördinates. To secure the *exact* form of Maxwell's equations in the local coordinate system was, in fact, the principal motive for the introduction of this system of field equations. Actually, however, a system of field equations constructed with primary regard to a law of conservation appears to be of deeper physical significance. This latter point of view is made the basis for the construction of a system of field equations in the present note—and the equations so obtained differ from those of Note I only by the appearance of terms quadratic in the quantities $h_{j,k}^i$. It would thus appear that we can carry over the interpretation of the h_α^i as electromagnetic potentials; doing this, we can say that Maxwell's equations hold approximately in the local coördinate system in the presence of weak electromagnetic fields.

Equations of Note I will be referred to by prefixing the numeral I before the number of the equation in question.

1. In the Einstein theory of gravitation the operation of forming the divergence of a tensor constituted an almost unavoidable generalization from the analogous operation of pre-relativity physics. When dealing with quantities of the nature of absolute invariants, however, as we shall do in the development of the present theory, it is by no means obvious how this operation can best be defined. However, actual investigation of the identities of the field theory points to a definition of the operation of divergence which is well adapted to a statement of the laws of conservation; the reasons for this particular definition will be apparent from the identities of the following sections.

DIVERGENCE RULE: *The formula*

$$\nabla_m T^i_{j\ldots m} = \sum_{m=1}^{4} e_m \left[T^i_{j\ldots m,m} + 2h^i_{m,r}\, T^r_{j\ldots m} \right]$$

defines the divergence of the invariant $T^i_{j\ldots m}$ *with respect to the index* m. Any covariant index, i.e., covariant under transformations of the fundamental vectors, can of course appear in place of the particular index m in the definition of the divergence operation. It should be observed that the term involving the absolute electromagnetic forces $2\, h^i_{j,k}$ in the above formula, corresponds to those which contained the components of affine connection in the divergence formula of the earlier theory of relativity. A generalization of the above formula to include invariants T with any number of contravariant indices could obviously be made but this is not necessary for the requirements of the present note. On the basis of the divergence rule we obtain the field equations from the following

POSTULATE OF THE UNIFIED FIELD. *The divergence of the absolute electromagnetic forces is equal to zero, i.e.,*

$$\boxed{\nabla_k\, h^i_{j,k} = 0}\ . \tag{1.1}$$

There are 16 equations in the system (1.1) for the determination of the 16 electromagnetic potentials h^i_α. By reference to the field equations in what follows we shall mean equations (1.1) rather than the corresponding equations of our previous note.

2. Before proceeding further we must derive certain special identities which we shall need in the following work; this will be done in as concise a manner as possible. We have

$$h^i_{j,k,l} = \frac{\partial h^i_{j,k}}{\partial x^\alpha}\, h^\alpha_l. \tag{2.1}$$

Hence

$$h^i_{j,k,l} = \tfrac{1}{2}\left[h^i_{j,kl} - k^i_{k,jl}\right] + h^i_{m,j} h^m_{k,l} + h^i_{m,k} h^m_{l,j} \tag{2.2}$$

when use in made I (4.5) and I (4.10). Interchanging k,l in (2.2) and adding, we obtain a set of identities which can be reduced to the form

$$h^i_{j,kl} = \tfrac{2}{3}[h^i_{j,k,l} + h^i_{j,l,k} + h^i_{m,k}\, h^m_{j,l} + h^i_{m,l} h^m_{j,k}]. \tag{2.3}$$

We observe that these latter identities constitute an inverse form of the identities (2.2) in that (2.2) expresses the $h^i_{j,k,l}$ in terms of the $h^i_{j,kl}$ while (2.3) gives $h^i_{j,kl}$ in terms of the $h^i_{j,k,l}$ invariants.

The invariants $h^i_{j,k,l}$ satisfy a set of identities

$$h^i_{j,k,l} + h^i_{k,l,j} + h^i_{l,j,k} = 2\,[h^i_{m,j}\,h^m_{k,l} + h^i_{m,k}\,h^m_{l,j} + h^i_{m,l}\,h^m_{j,k}] \qquad (2.4)$$

which is easily deduced from (2.2).

Let us denote the quantities $\Lambda^\alpha_{\beta\gamma\delta}$ by $\lambda^i_{j,k,l}$ with reference to a system of local coördinates and define the invariant $\mathfrak{A}^i_{jklm}$ by the formula

$$\mathfrak{A}^i_{jklm} = \left(\frac{\partial \lambda^i_{jkl}}{\partial z^m}\right)_{z=0}.$$

(See Sect. 2 in Note I.) Now transform equations I (4.6) to local coordinates, differentiate, and evaluate at the origin of the local system; we thus obtain a set of identities which can be written

$$h^i_{j,kl,m} = h^i_{j,klm} - \mathfrak{A}^i_{jklm} + \tfrac{1}{2}\,h^i_{j,t}\,h^t_{m,kl} + \tfrac{1}{2}\,h^i_{l,k}\,h^t_{m,jl} + \tfrac{1}{2}\,h^i_{l,l}\,h^t_{m,jk} + h^t_{m,j}\,h^i_{t,kl} + h^t_{m,k}\,h^i_{j,tl} + h^t_{m,l}\,h^i_{j,kt}. \qquad (2.5)$$

Interchanging the indices j,k in (2.5) and subtracting the resulting identities from (2.5) we obtain

$$h^i_{j,kl,m} - h^i_{k,jl,m} = h^i_{j,klm} - h^i_{k,jlm} + h^i_{j,t}\,h^t_{m,kl} + h^i_{l,k}\,h^t_{m,jl} + [h^t_{m,j}\,(h^i_{l,kl} - h^i_{k,tl}) + h^t_{m,k}\,(h^i_{j,tl} - h^i_{l,jl}) + h^t_{m,l}\,(h^i_{j,kl} - h^i_{k,jl})]. \qquad (2.6)$$

To the identities (2.6) we now add those two sets of identities which result from (2.6) by permutating the indices k,l,m cyclically. By a suitable arrangement of terms the set of identities so obtained can be given the form

$$8h^i_{j,klm} = 3\,[h^i_{j,kl,m} + h^i_{j,lm,k} + h^i_{j,mk,l} + h^i_{j,m}\,h^t_{t,kl} + h^i_{j,l}\,h^t_{t,mk} + h^i_{j,k}\,h^t_{t,lm}] + h^i_{t,k}\,h^t_{j,lm} + h^i_{t,l}\,h^t_{j,km} + h^i_{t,m}\,h^t_{j,kl}. \qquad (2.7)$$

These identities express the invariants $h^i_{j,klm}$ in terms of the invariants $h^i_{j,kl,m}$ plus invariants of lower order in the derivatives of the electromagnetic potentials.

To obtain the inverse form of the set of identities (2.7) we deduce the formula

$$\mathfrak{A}^i_{jklm} = -\tfrac{1}{3}\,h^i_{m,jkl} + \tfrac{1}{6}\,(h^i_{t,j}\,h^t_{m,kl} + h^i_{t,k}\,h^t_{m,lj} + h^i_{t,l}h^t_{m,jk})$$

which we use to eliminate the invariants $\mathfrak{A}^i_{jklm}$ from (2.5). The result of this elimination is a set of identities which can be put into the form

$$3h^i_{j,kl,m} = 3h^i_{j,klm} + h^i_{m,jkl} + 2\,h^i_{j,t}\,h^t_{m,kl} + h^i_{t,k}\,h^t_{m,jl} + h^i_{t,l}\,h^t_{m,jk} + 3\,(h^t_{m,j}\,h^i_{t,kl} + h^t_{m,k}\,h^i_{j,tl} + h^t_{m,l}\,h^i_{j,kl}). \qquad (2.8)$$

Finally the set of identities

$$5h^i_{j,kl,m} = 3\,(h^i_{j,lm,k} + h^i_{j,mk,l}) + h^i_{m,kl,j} + h^i_{m,lj,k} + h^i_{m,jk,l} + h^i_{i,l}$$
$$(3\,h^i_{m,jk} + h^i_{j,km}) + h^i_{i,k}\,(3\,h^i_{m,jl} + h^i_{j,lm}) \qquad (2.9)$$
$$+ 5\,h^i_{j,i}\,h^i_{m,kl} + h^i_{i,m}\,h^i_{j,kl} + 6\,h^i_{m,j}\,h^i_{i,kl} + h^i_{m,k}\,(8\,h^i_{j,il} + h^i_{i,jl})$$
$$+ h^i_{m,l}\,(8\,h^i_{j,ki} + h^i_{i,jk}) + 3\,(h^i_{j,l}\,h^i_{i,mk} + h^i_{j,k}\,h^i_{i,lm})$$

can be obtained by eliminating the invariants $h^i_{j,klm}$ between the identities (2.7) and (2.8).

3. The identities of Sect. 2 can be used to deduce identities of especial interest for the field theory. By use of (2.2) it readily follows that

$$\nabla_k\,h^i_{j,k} = \tfrac{3}{4}\sum_{k=1}^{4} e_k\,(h^i_{j,kk} + 4\,h^i_{r,k}\,h^r_{k,j}) \qquad (3.1)$$

identically. Hence

$$\boxed{\sum_{k=1}^{4} e_k\,(h^i_{j,kk} + 4\,h^i_{r,k}\,h^r_{k,j}) = 0} \qquad (3.2)$$

constitutes a system of equations completely equivalent to the field equations (1.1).

Now put $l = k$ and $m = j$ in (2.9), then multiply these equations through by e_je_k and sum on the two repeated indices. This gives

$$\sum_{j=1}^{4}\sum_{k=1}^{4} e_j\,e_k\,(h^i_{j,kk,j} + h^i_{r,j}\,h^r_{j,kk} + 2\,h^r_{k,j}\,h^i_{j,kr}) = 0 \qquad (3.3)$$

identically. We next consider the set of identities

$$2\sum_{j=1}^{4}\sum_{k=1}^{4} e_j\,e_k\,h^r_{k,j}\,h^i_{j,kr}$$
$$= 3\sum_{j=1}^{4}\sum_{k=1}^{4} e_j\,e_k\,h^i_{j,m}\,(h^m_{j,kk} + \tfrac{8}{3}\,h^m_{r,k}\,h^r_{k,j}) + 4\sum_{j=1}^{4}\sum_{k=1}^{4} e_j\,e_k\,(h^i_{r,k}\,h^r_{k,j})_{,j}$$

which we use to eliminate the last set of terms from (3.3). As a result we obtain the set of four identities

$$\sum_{i=1}^{4} e_j\Big\{\Big[\sum_{k=1}^{4} e_k\,(h^i_{j,kk} + 4\,h^i_{r,k}\,h^r_{k,j})\Big]_{,j} + 2\,h^i_{j,m}$$
$$\Big[\sum_{k=1}^{4} e_k\,(h^m_{j,kk} + 4\,h^m_{r,k}\,h^r_{k,j})\Big]\Big\} = 0.$$

By (3.1) this last set of identities can be given the form

$$\boxed{\nabla_j\,\nabla_k\,h^i_{j,k} = 0}, \qquad (3.4)$$

i.e., *the divergence of the left member of* (1.1) *vanishes identically.* The identities (3.4) are the mathematical counterpart of the laws of conservation of the physical world.[2]

4. If we fail to take into account the conditions imposed by the field equations on the structure of space, then the number of independent invariants $h^i_{j,kl_1\ldots l_r}$ of order $r+1$ in the derivatives of the electromagnetic potentials h^i_α, i.e., the number of arbitrary values $(h^i_{j,kl_1\ldots l_r})_Q$ which these invariants can assume at a point Q, is

$$16\,K(4,\,r+1) - 4K(4,\,r+2),$$

where $K(p,q)$ denotes the number of combinations with repetitions of p things taken q at a time; this is an immediate consequence of the fact that I (4.7) and I (4.9) constitute a complete set of identities. We shall denote the above number by $N(r+1)$ and observe that we can write

$$N(r+1) = 12K(4,r) + 8K(3,r) + 4K(2,r)$$

It can easily be shown that the number of independent quantities $(h^i_{j,k,l_1\ldots l_r})_Q$ is likewise given by $N(r+1)$. To do this we transform the expression for $h^i_{j,k}$ given by 1 (4.5), to a system of local coördinates, differentiate repeatedly, and evaluate at the origin of the system. We thereby obtain a system of equations of the form

$$h^i_{j,k,l_1\ldots l_r} = \tfrac{1}{2}\,[h^i_{j,kl_1\ldots l_r} - h^i_{k,jl_1\ldots l_r}] + \bigstar, \qquad (4.1)$$

where the $\bigstar$ is used to denote terms of lower order than those which have been written down explicitly. Now let P denote the operations of holding j fixed, permuting the indices $kl_1\ldots l_r$ cyclically, and adding the resulting terms. Then

$$2\,P(h^i_{j,k,l_1\ldots l_r}) = P\,(h^i_{j,kl_1\ldots l_r}) - P\,(h^i_{k,jl_1\ldots l_r}) + \bigstar$$

from (4.1) or

$$(r+2)\,h^i_{j,kl_1\ldots l_r} = 2\,\mathrm{P}\,(h^i_{j,k,l_1\ldots l_r}) + \bigstar. \qquad (4.2)$$

In view of (4.1) and (4.2) it follows that if we consider the quantities

$$h^i_\alpha;\;\; h^i_{j,k};\;\; \ldots;\;\; h^i_{j,kl_1\ldots l_{r-1}}$$

to have fixed values at Q, the number of independent quantities $(h^{ki}_{j,k,l_1\ldots l_r})_Q$ is equal to the number of independent quantities $(^{i}_{j,kl_1\ldots l_r})_Q$, i.e., the number $N(r+1)$.

By making use of the above result it can be shown that $N(r+1)$ gives the number of arbitrary partial derivatives of the rth order of the quantities $h^i_{j,k}$ at the point Q. The reader can easily work out the details of this demonstration.

5. When account is taken of the conditions imposed by the field equa-

tions, the number of arbitrary partial derivatives of the quantities $h^i_{j,k}$ at the point Q is decreased. A *lower bound* to this number can, however, be deduced in the following manner. Taking the absolute derivative of the left member of (1.1) we obtain

$$(\nabla_k h^i_{j,k})_{,l} = 0. \tag{5.1}$$

By (3.4) four of the equations of the set (5.1) are linearly dependent on the remaining equations (5.1) and the field equations (1.1). In fact,

$$(\nabla_k h^i_{l,k})_{,l} = \ldots,$$

where the dots have been used to denote a linear expression in certain of the left members of (5.1) plus a linear expression in the quantities in the left members of (1.1) with coefficients equal to the absolute electromagnetic forces. Assuming that the $h^i_{j,k}$ and all their partial derivatives to those of order r ($\geqq 1$) inclusive have fixed values at the point Q, it follows that the number of partial derivatives of the quantities $h^i_{j,k}$ of order $(r+1)$ at Q, whose values are determined as a consequence of equations (1.1), is *at most* equal to

$$16K(4,r) - 4K(4,r-1). \tag{5.2}$$

Hence the arbitrary derivatives of the $h^i_{j,k}$ of order $r+1$ at Q cannot be less than the difference of $N(r+2)$ and (5.2), i.e.,

$$16K(3,r+1) + 8K(2,r+1);$$

for $r = 0$ this expression likewise gives the number of arbitrary derivatives of the first order of the $h^i_{j,k}$. In our next note we shall apply the above lower bound to the problem of constructing the general existence theorem for the field equations.

[1] These PROCEEDINGS, **16**, 761–776 (1930).

[2] If we replace (1.1) by

$$\nabla_k h^i_{j,k} = D^i_j,$$

where D^i_j is the analogue of the vector of charge and current density of the classical theory, then (3.4) yields

$$\nabla_j D^i_j = 0.$$

These latter equations are the analogue of the equation of the classical theory which shows that electric charge is conserved. We have purposely failed to introduce the invariants D^i_j of unknown structure into our field theory as it is our wish to investigate the extent to which the physical world can be described by equations of the type (1.1).

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